MODELS FOR TENSION STIFFENING AND DEFLECTIONS OF GFRP-RC

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Outline

- Introduction
- Problem definition
- Methodology
  - Direct tension studies to quantify Tension stiffening effect
- Modelling tension behaviour GFRP-RC
- Modelling Deflections
- Conclusions and remarks

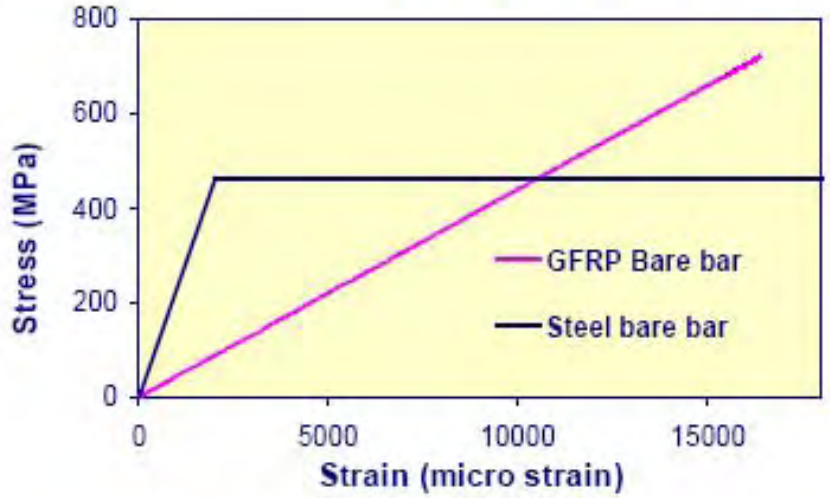
Introduction: GFRP RC in Construction

Corrosions of Steel reinforcements

Use of GFRP for bridge deck construction
(Franklin county bridge Virginia)

GFRP bars

Steel bars



Stiffness of GFRP compared to Steel

Tension Stiffening in Design Codes

In ACI

$$\Delta = \frac{kPl^3}{EI_{eff}}$$

Branson's equation for I_{eff}

$$I_{eff} = I_g\left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]$$

ACI 440 approach

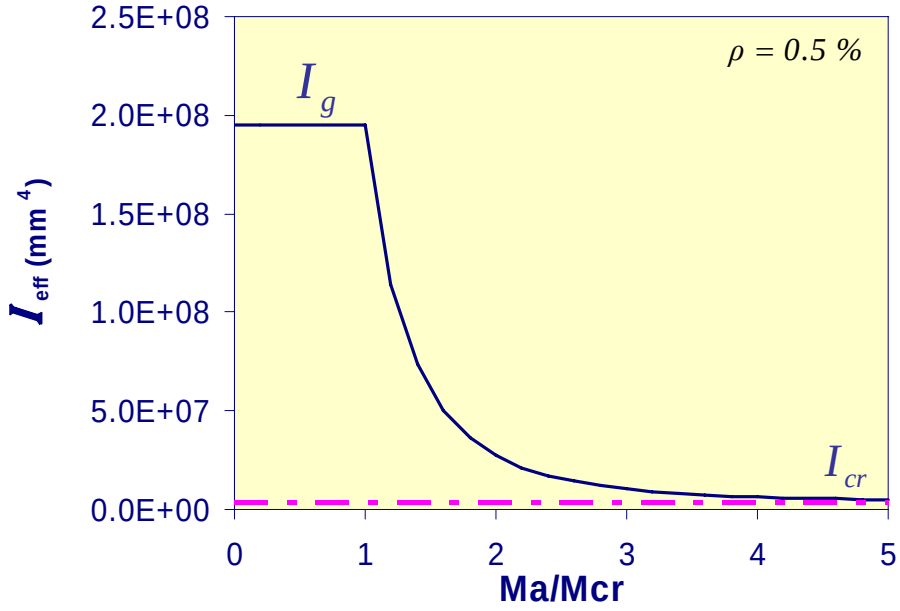
$$I_{eff} = I_g\beta_d\left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]$$

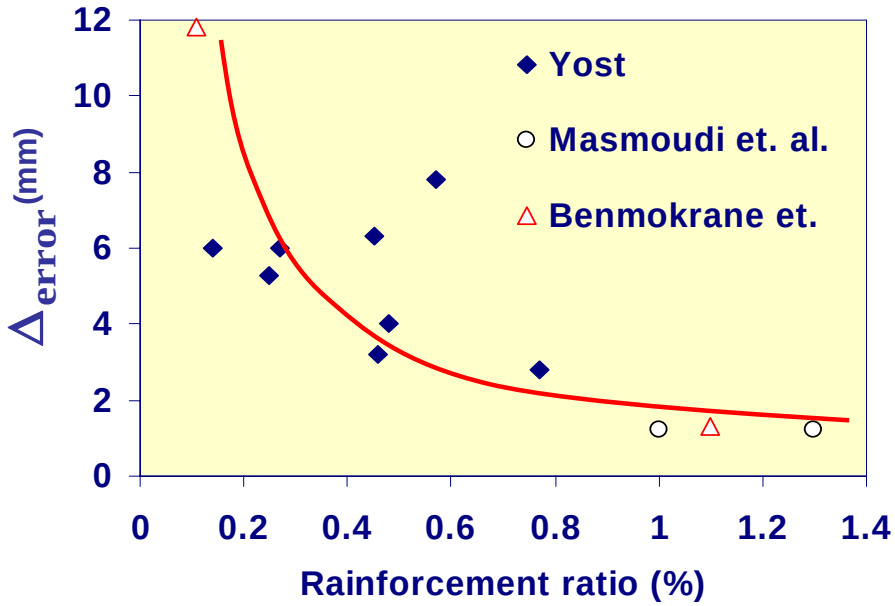
$$\beta_d = \alpha_b\left[\frac{E_f}{E_s}+1\right]$$

$$\alpha_b = 0.5$$

Δ_{error} = Experimental Deflection minus the Deflection by Branson's Equation, both at service level (50% ultimate load)

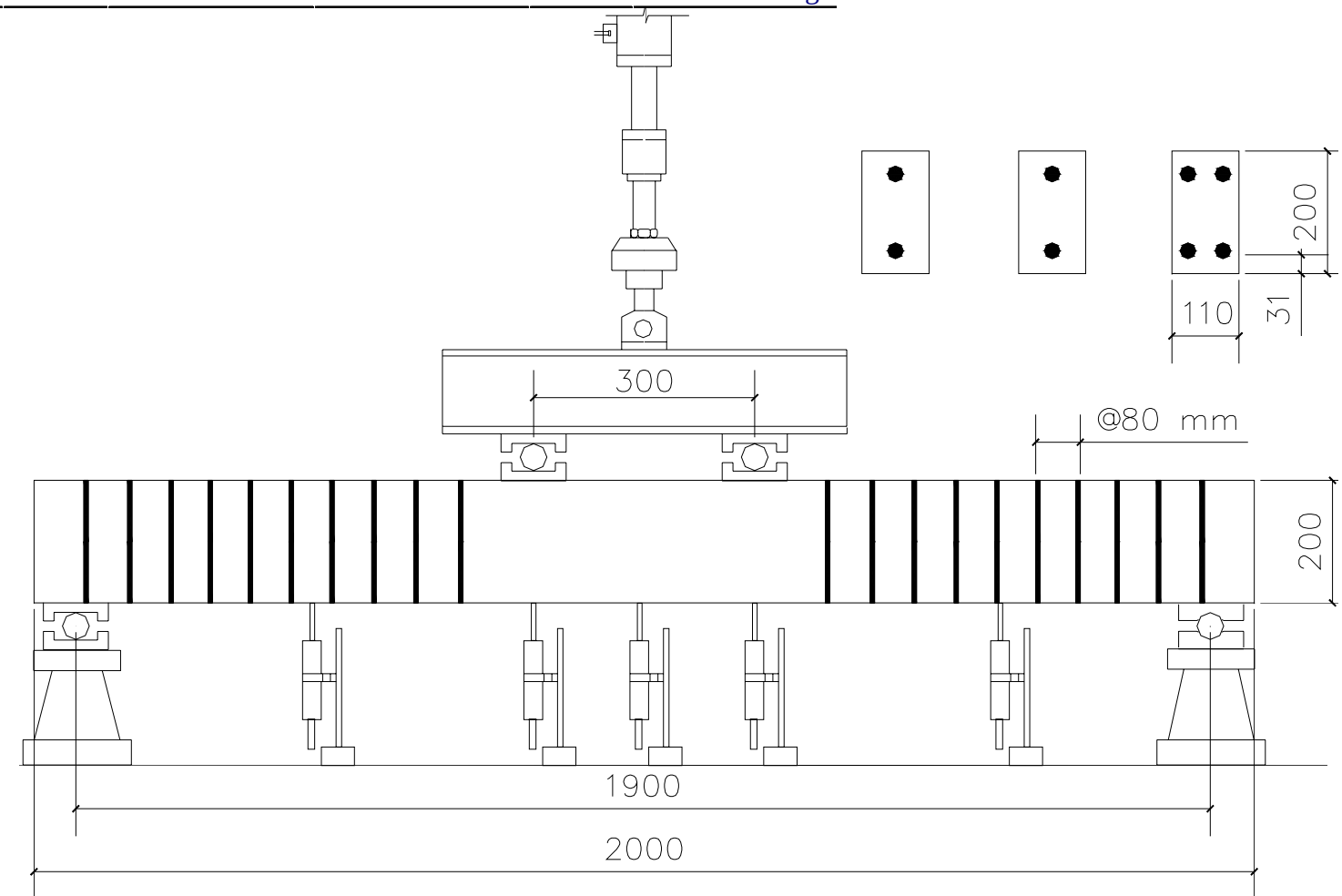
(courtesy Toutanji et al. (2003), Construction and Building Material)





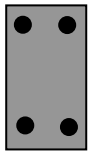
Test on deflections –beam series

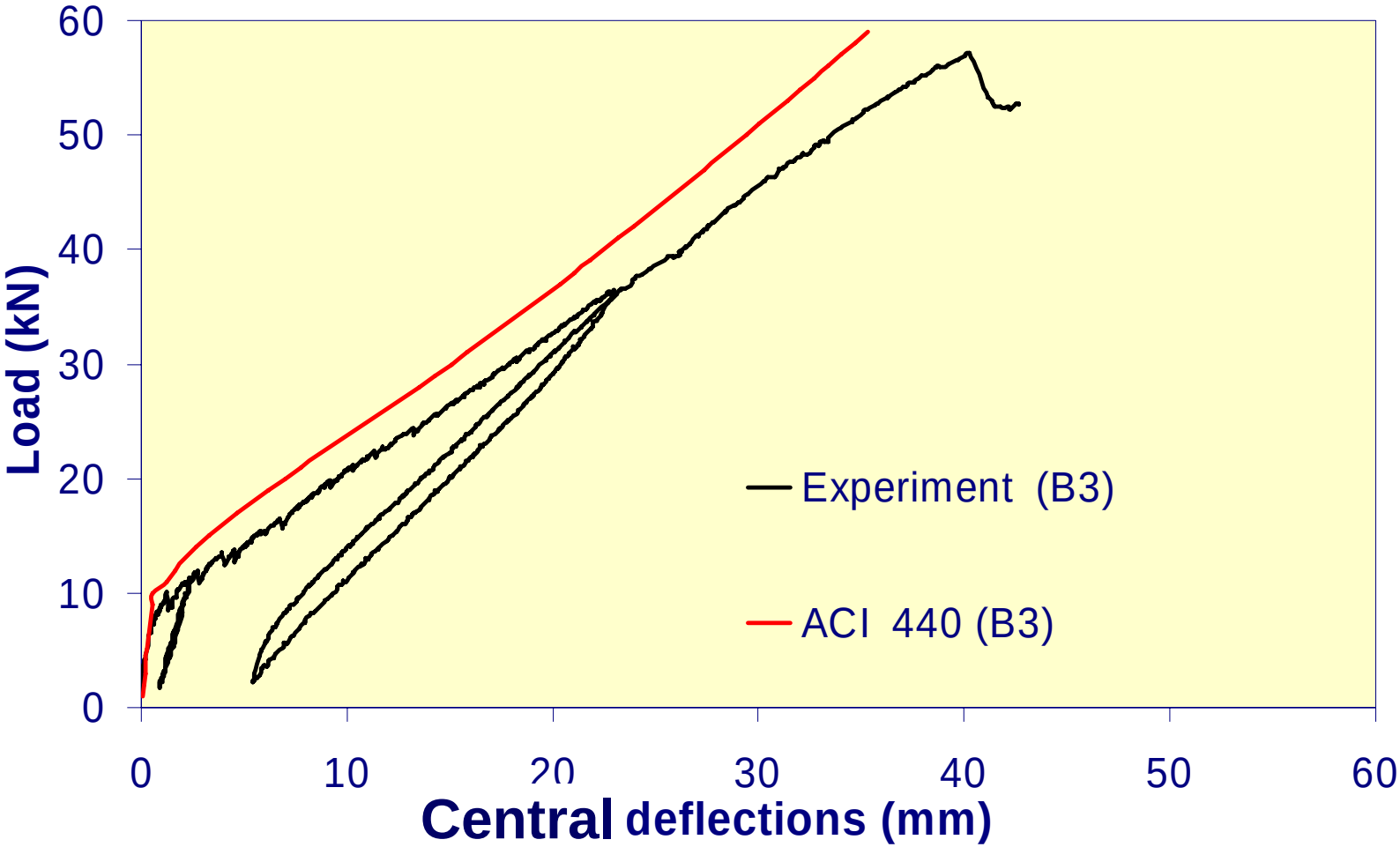
Rebar type	Beam	Cover (mm)	Reinforcement	Rein. Ratio $\rho = As / bh$ (%)	f_{cu} (MPa)	Failure mode
GFRP	B1	31	1∅12.7 mm	0.57	91	Bar failure
	B2	31	1∅12.7 mm	0.57	46	Con. Crushing
	B3	31	2∅12.7 mm	1.15	46	Con. Crushing



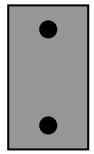
Tests on deflections –beam series (B3)

$\rho = 1.15$

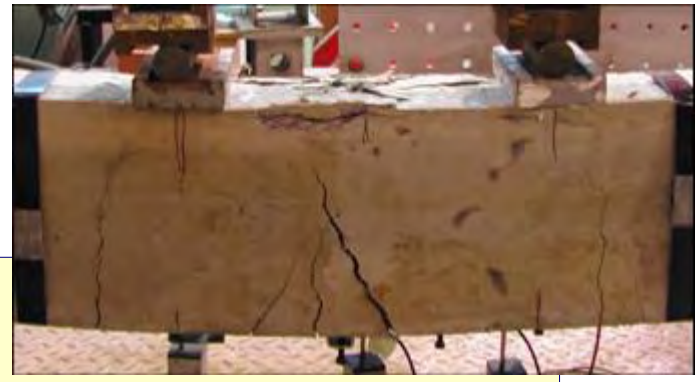


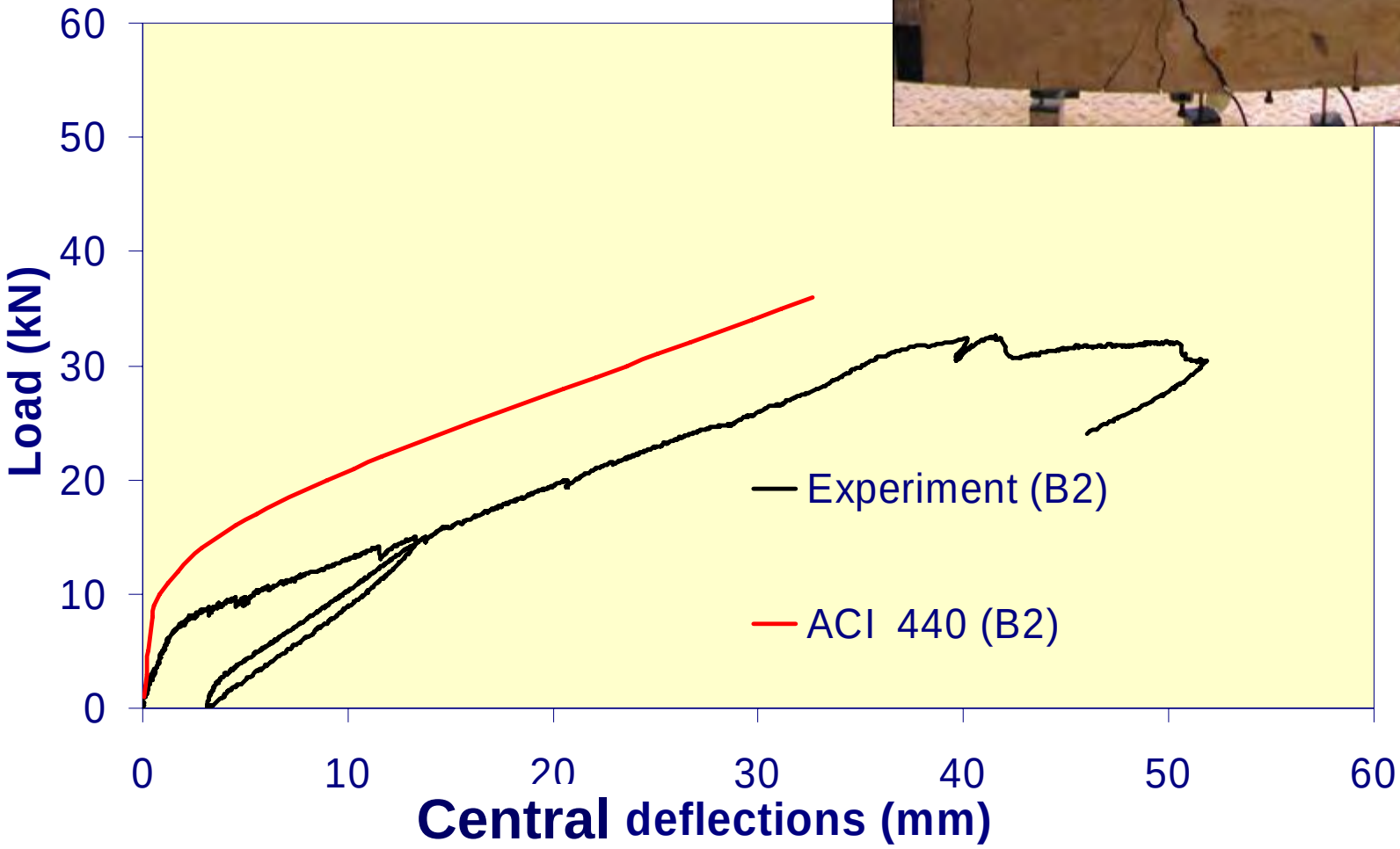


Tests on deflections –beam series (B2)

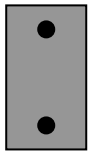


$\rho = 0.57$

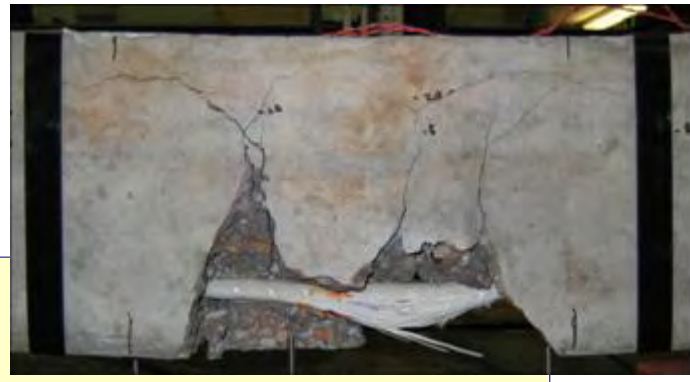


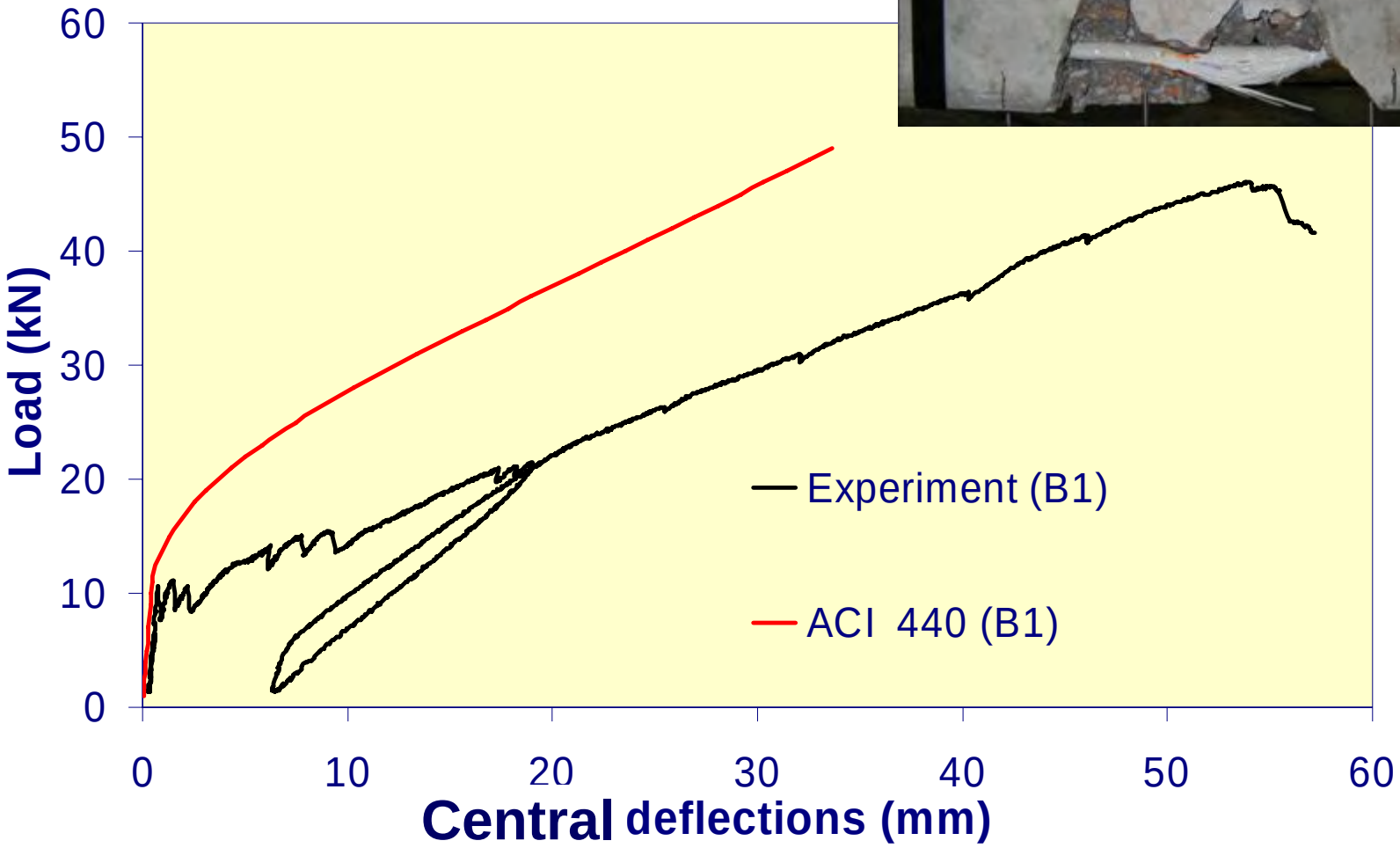


Tests on deflections –beam series (B1)



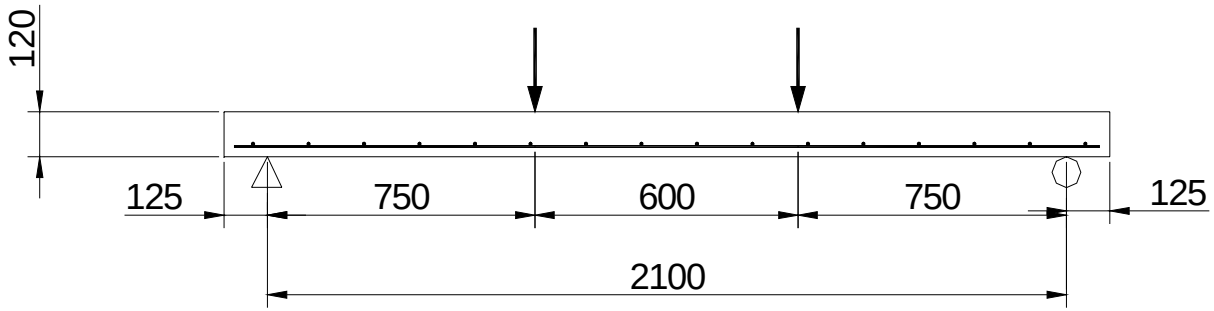
$\rho = 0.57$

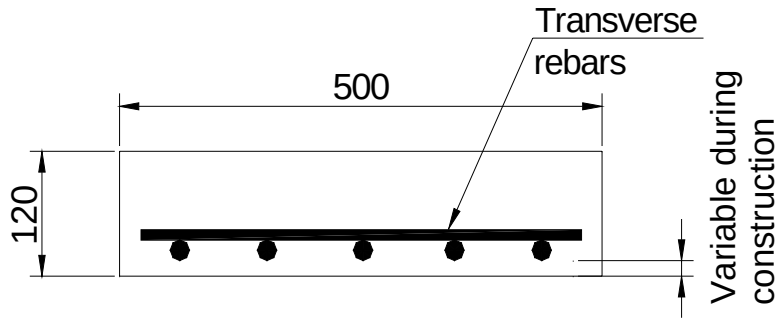




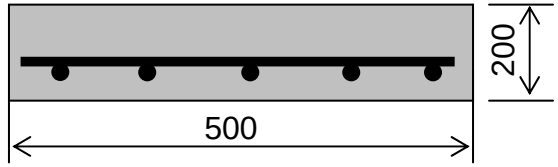
Tests on deflections –Slab series

Rebar type	Slab	Cover, (mm)	Reinforcement	Reinforcement Ratio $\rho = As / bh$ (%)	Concrete Cylinder strength "MPa"
GFRP	S1	27.5	5∅6mm	0.24	43
	S2	31	5∅9.53mm	0.59	39
	S3	40	5∅19.05mm	2.38	39

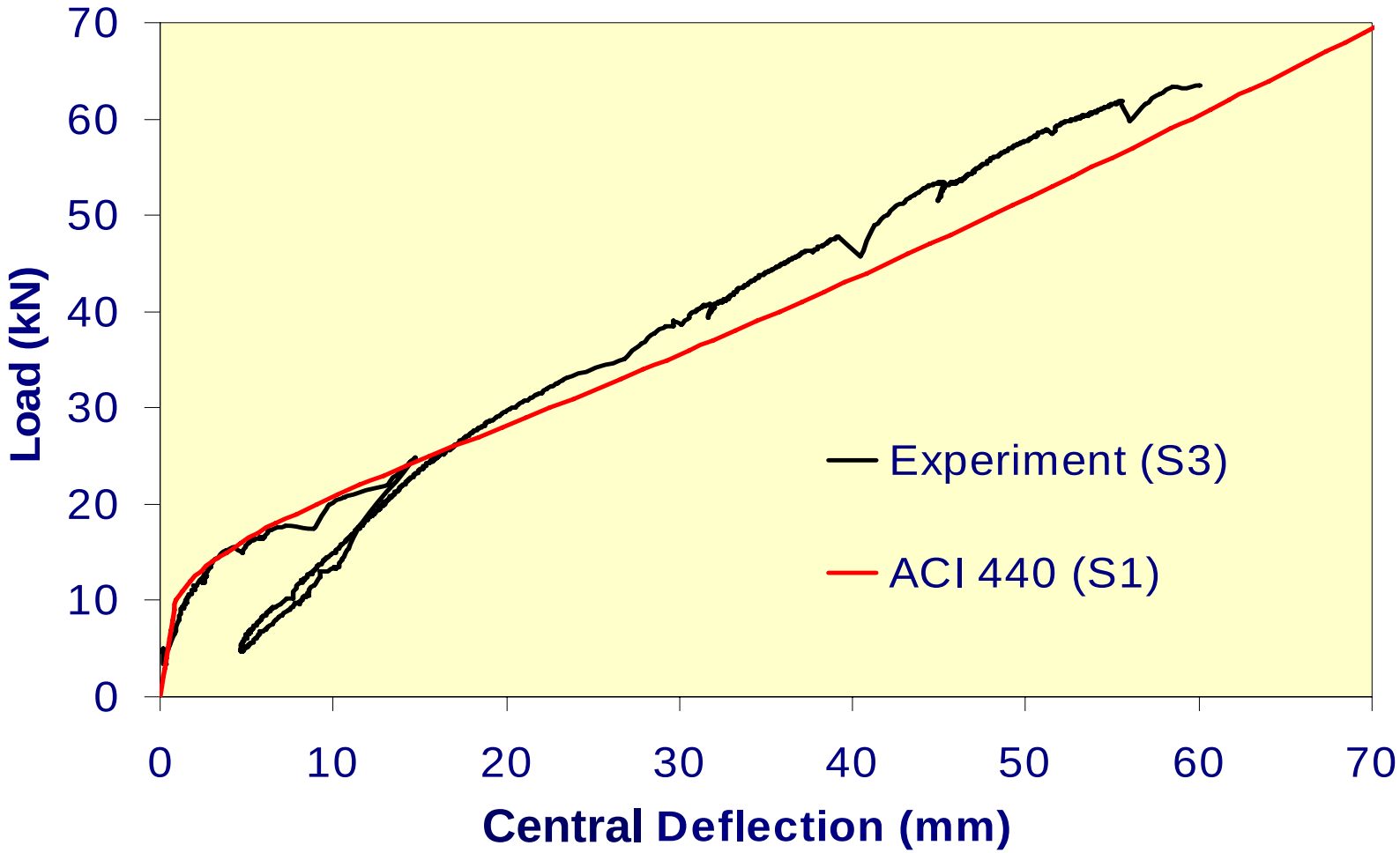




Test on deflections –beam series (S3)

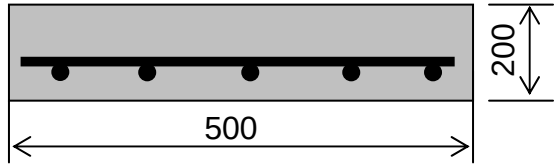


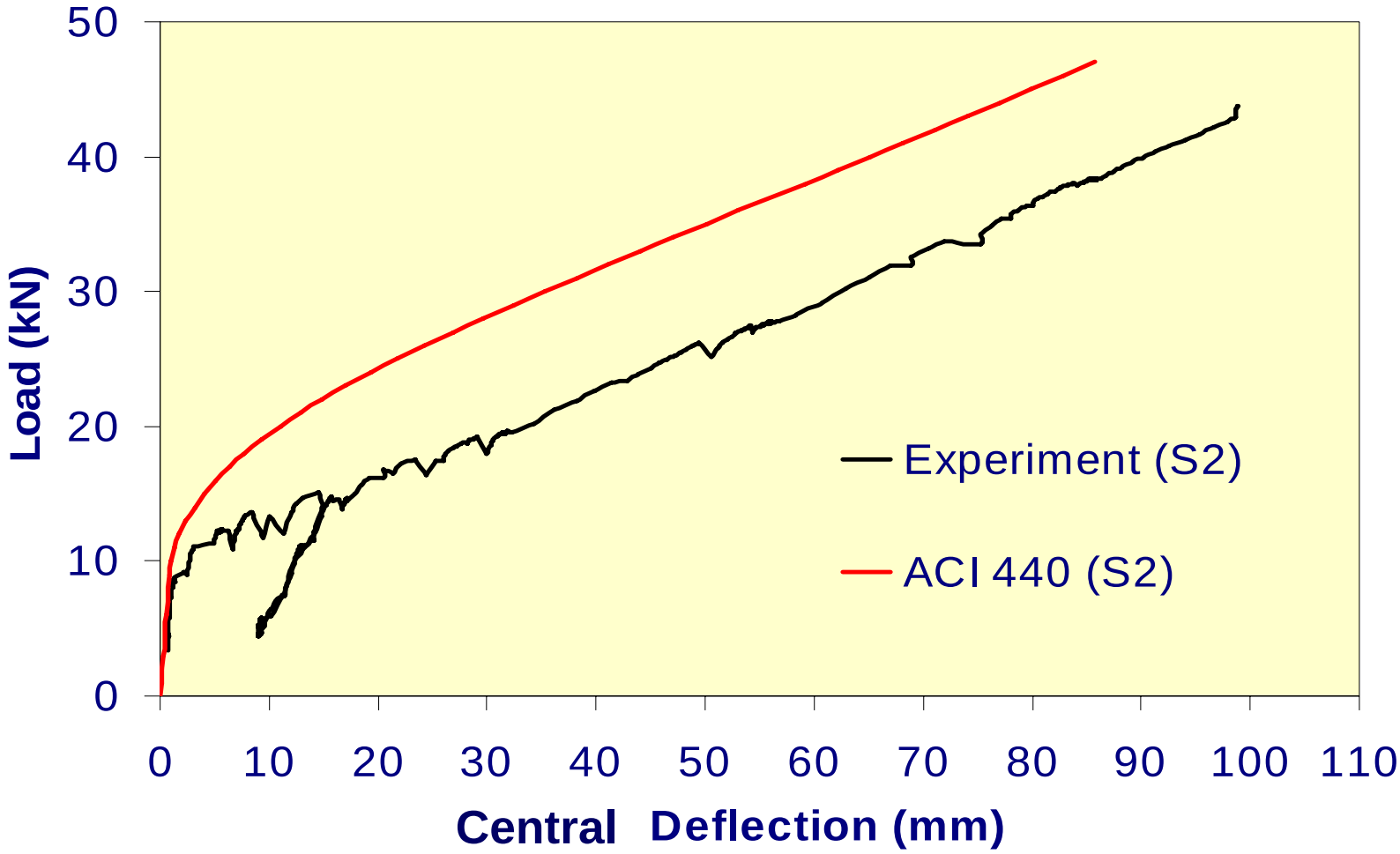
$\rho = 2.38$



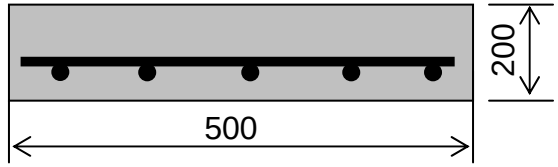
Test on deflections –beam series (S2)

$\rho = 0.59$

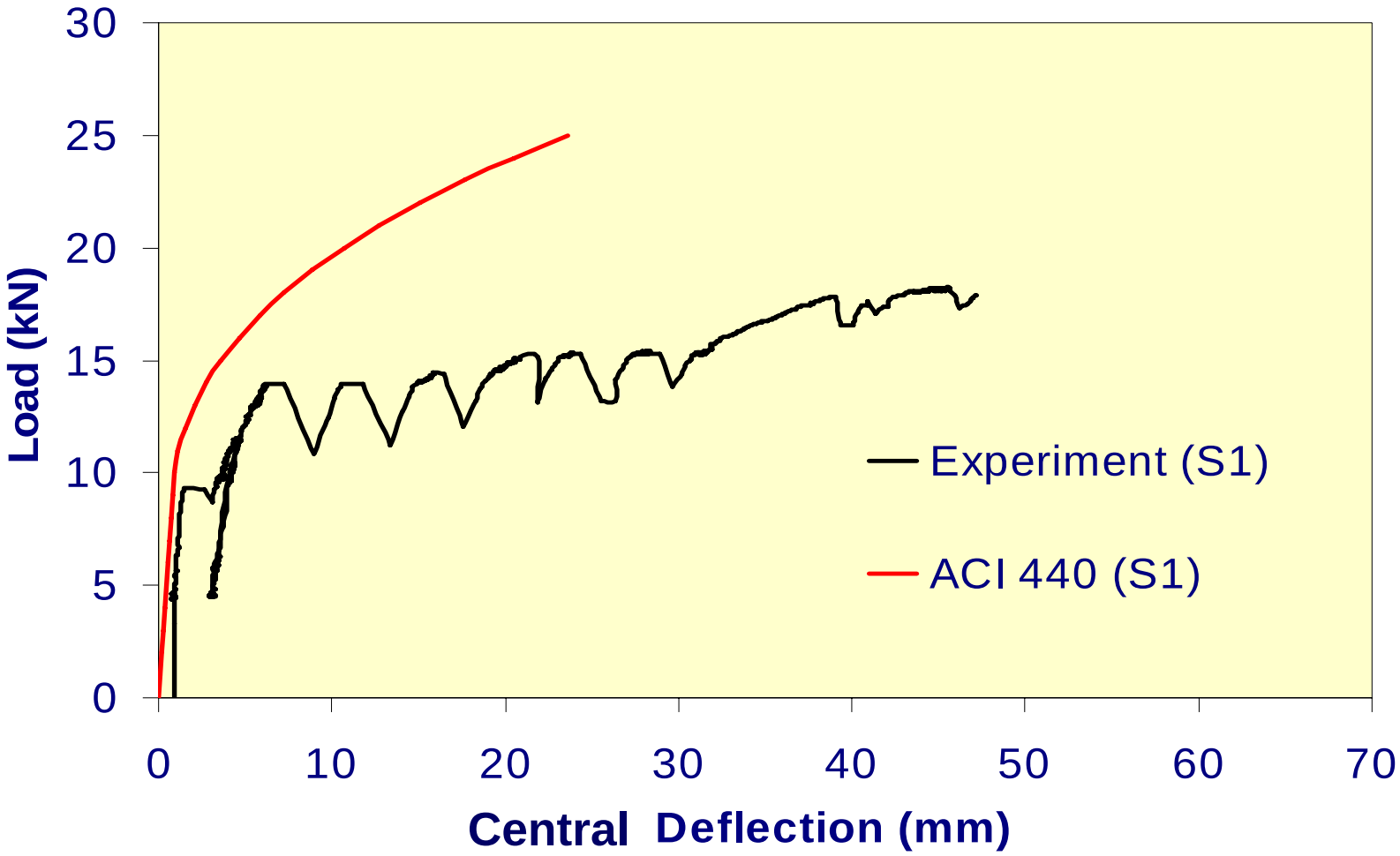




Test on deflections –beam series (S1)



$\rho = 0.24$



Tension Stiffening for FRP Design

- **No general agreement on tension stiffening**

$$I_{eff} = I_g\left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]$$ *ACI Branson's*

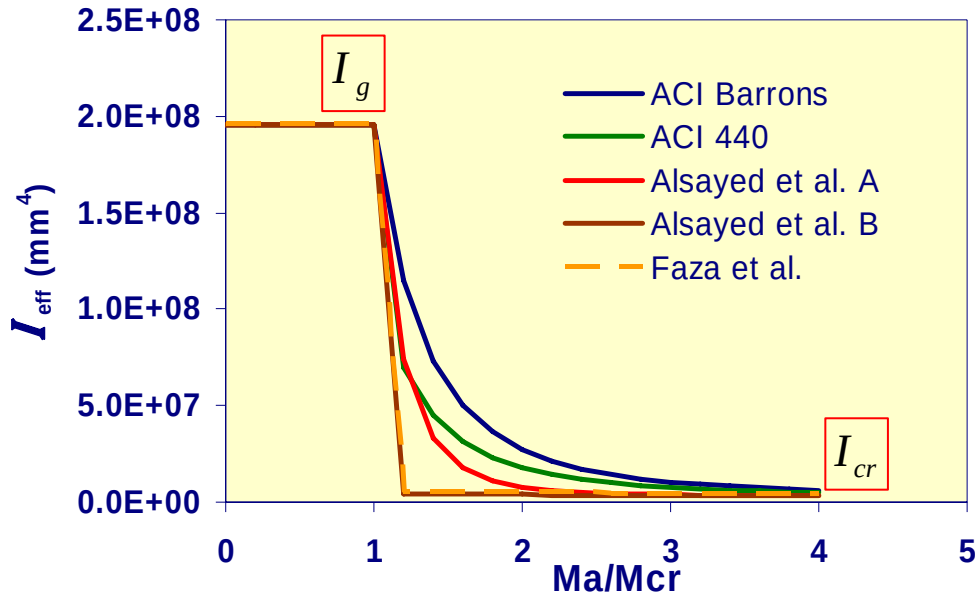
$$I_{eff} = I_g\beta_d\left(\frac{M_{cr}}{M_a}\right)^3 + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^3\right]$$ *ACI 440*

$$I_m = \frac{23I_{cr}I_e}{8I_{cr}+15I_e}, \quad I_e = ACI \quad I_{eff}$$ *Faza et. al B*

$$I_{eff} = I_g\left(\frac{M_{cr}}{M_a}\right)^{5.5} + I_{cr}\left[1-\left(\frac{M_{cr}}{M_a}\right)^{5.5}\right]$$ *Alsayed et. al A*

$$1 < \frac{M_a}{M_{cr}} < 3 \Rightarrow I_{eff} = I_{cr}\left[1.40 - \frac{2}{15}\left(\frac{M_a}{M_{cr}}\right)\right]$$ *Alsayed et. al B*

$$I_{eff} = I_{cr}$$



Research Approach

- Study the tension stiffening effect at fundamental level
- Develop a suitable way to incorporate tension stiffening in deflection

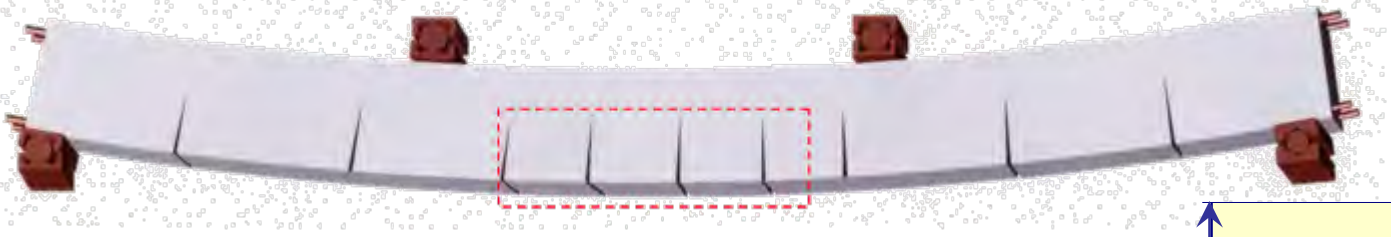
Studying tension stiffening alone was important at the time as it is necessary for Modelling GFRP-RC using FE Method based on smeared crack approach.

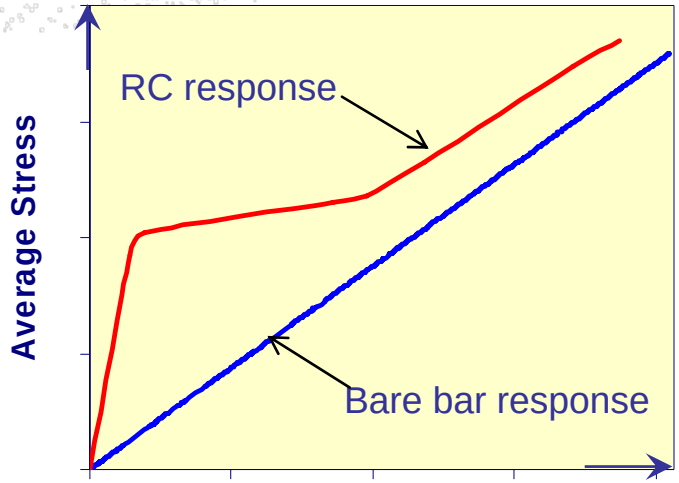
Definition- Tension Stiffening

Tension stiffening of concrete is defined as:

the ability of concrete to carry tension between cracks and provide extra stiffness for RC in tension.

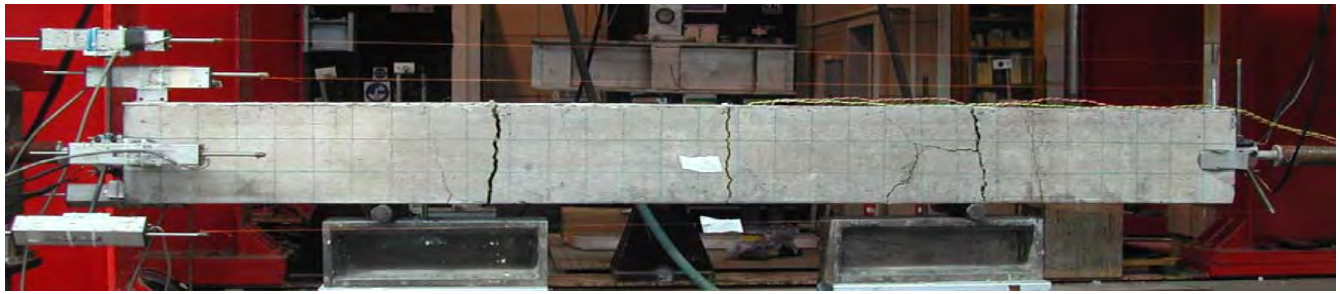
- **Serviceability often governs GFRP-RC design**
- **Tension stiffening is very important for the determination of deflections and crack widths at low load levels**

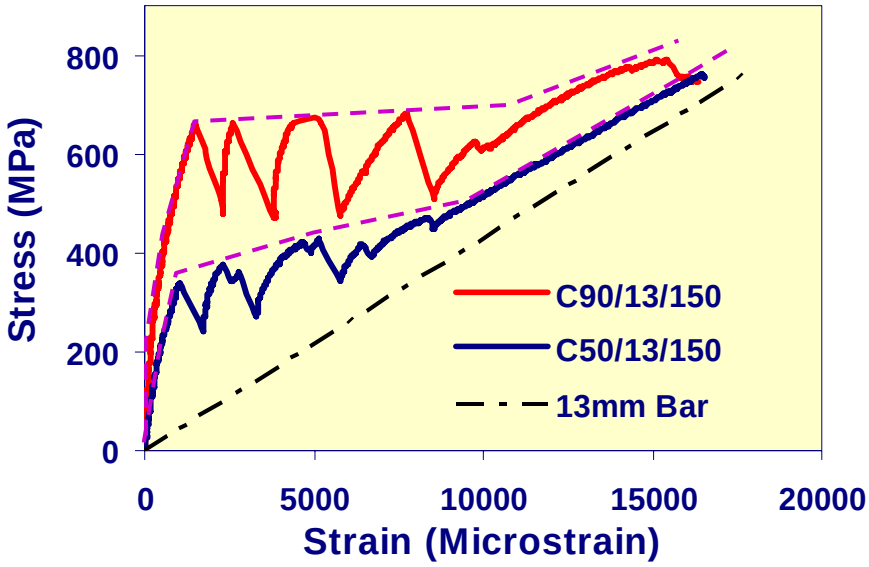




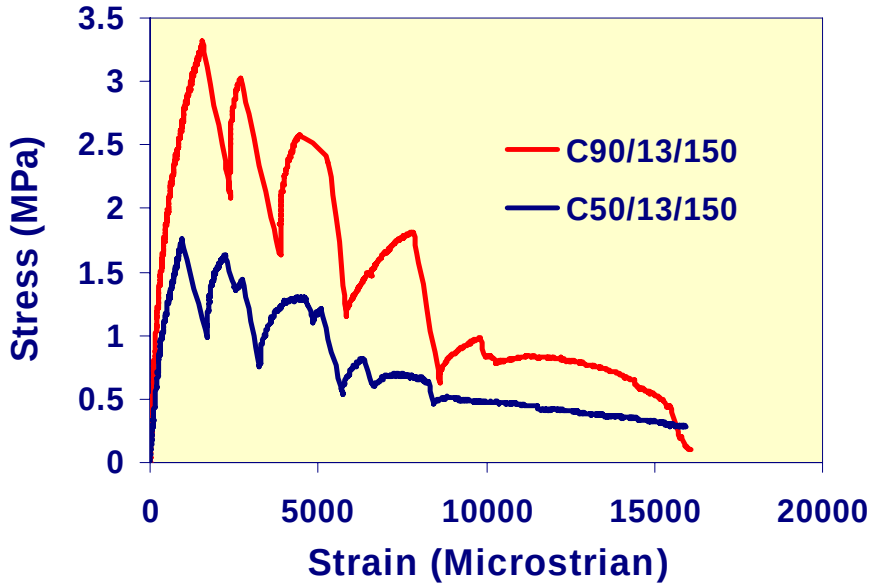
Tension stiffening effect

Strain Softening Behaviour Concrete





Test results bar stress Vs overall strain

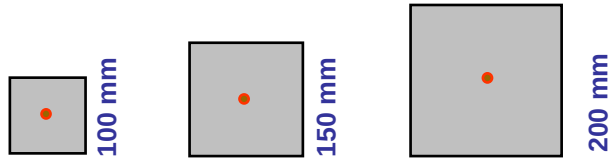


Average stress strain behaviour of concrete

The University Of Sheffield.

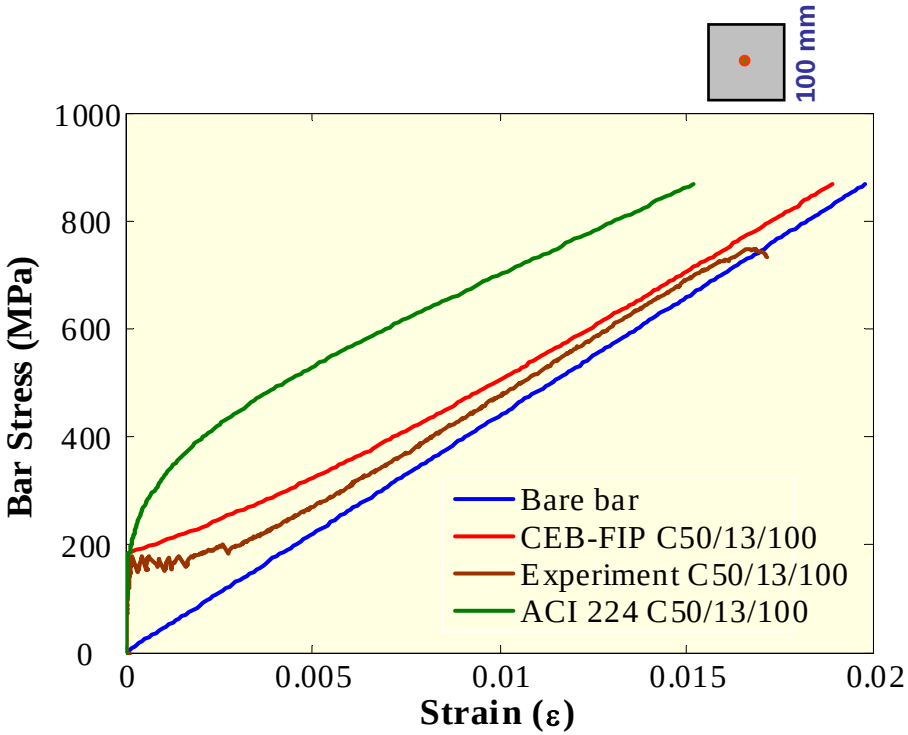
Parametric Study

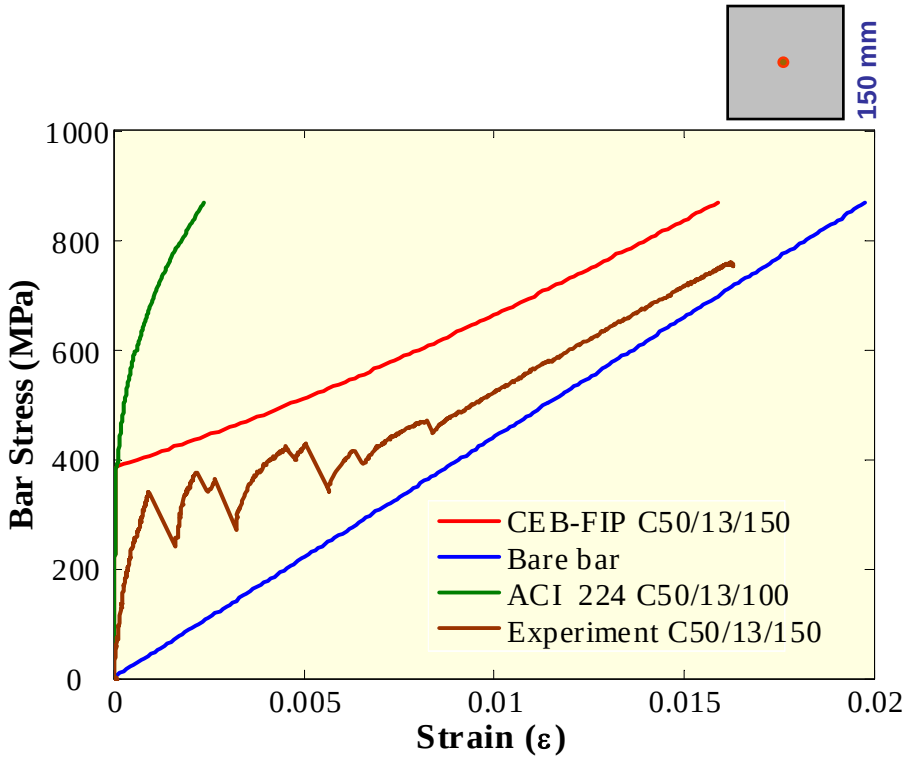
FRPRCS-8



Specimen	Concrete strength (f_c')	Bar Diameter (ϕ)	Dimension *b*×*d*×*l*	Reinforcement ratio (ρ)
C50/13/100	46	12.7	100×100×1500	1.26
C50/13/150	46	12.7	150×150×1500	0.56
C50/13/200	46	12.7	200×200×1500	0.32
C90/13/100	91	12.7	100×100×1500	1.26
C90/13/150	91	12.7	150×150×1500	0.56
C50/19/150	46	19.1	150×150×1500	1.27
C50/19/200	46	19.1	200×200×1300	0.72
C90/19/150	46	19.1	150×150×1300	1.27
C90/19/200	91	19.1	200×200×1300	0.72
C50/19/200N	91	19.1	200×200×1300	0.72

Prediction of tension stiffening effect - Code based approach





Reduced cross sectional area

Original

$$A_e = \left[\frac{P_{cr}}{P_a}\right]^3 A_g + \left[1 - \left(\frac{P_{cr}}{P_a}\right)^3\right] A_{cr}$$

Modified to account for weak FRP bond

$$A_e = \left[\frac{P_{cr}}{P_a}\right]^3 \beta_d A_g + \left[1 - \left(\frac{P_{cr}}{P_a}\right)^3\right] A_{cr}$$

→ ACI

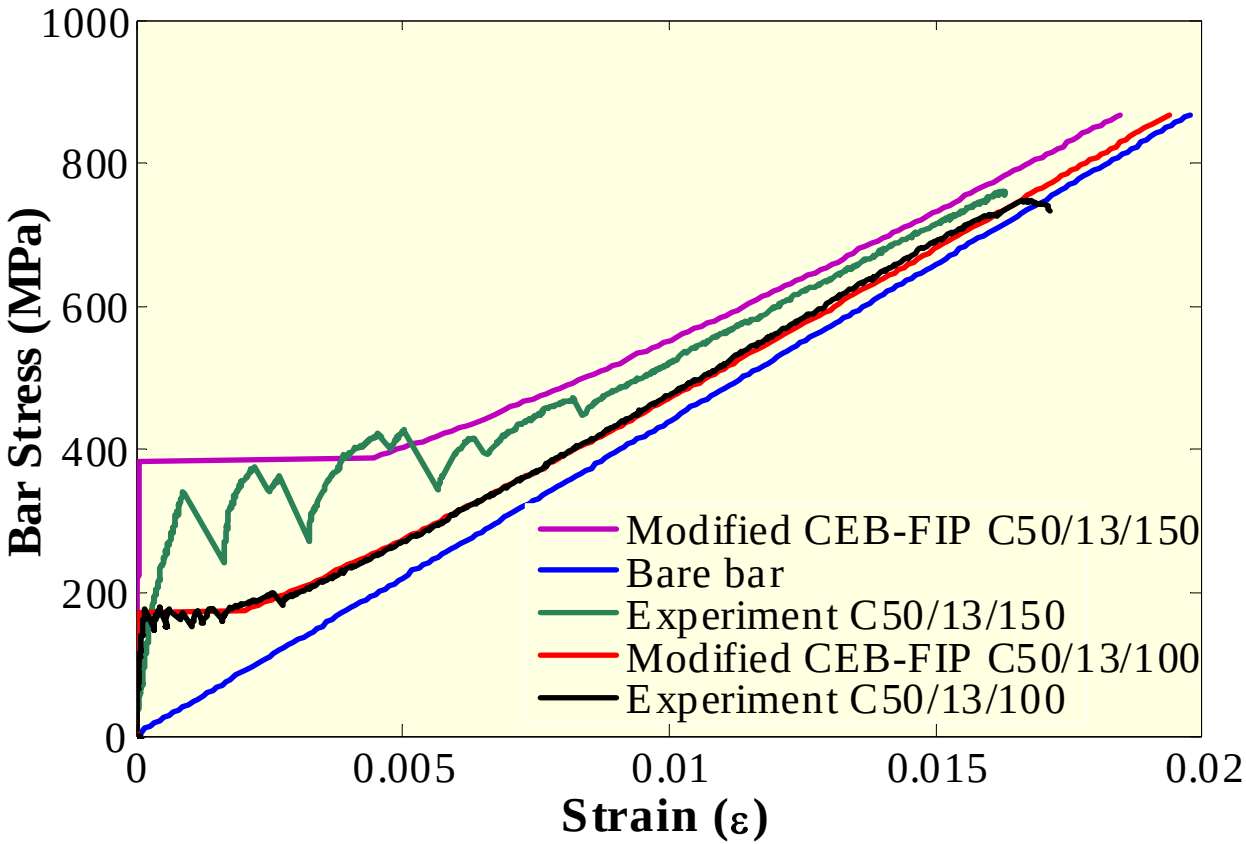
Composite strain for given bar strain

$$\varepsilon_m = \varepsilon_s \left[1 - K\left(\frac{f_{scr}}{f_f}\right)^2\right]$$

$$f_{scr} = \frac{P_{cr}}{A_f} = f_t'\left(\frac{1}{\rho} - 1 + n_f\right)$$

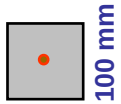
→ CEB

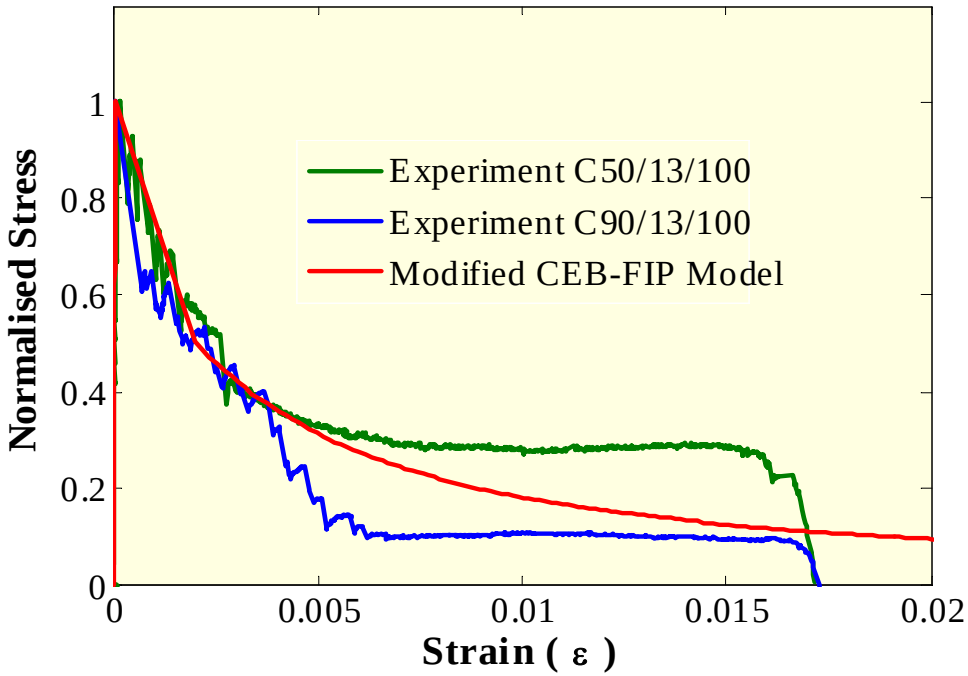
Model for Tension Stiffening effect of GFRP-RC

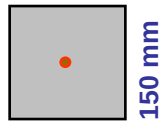


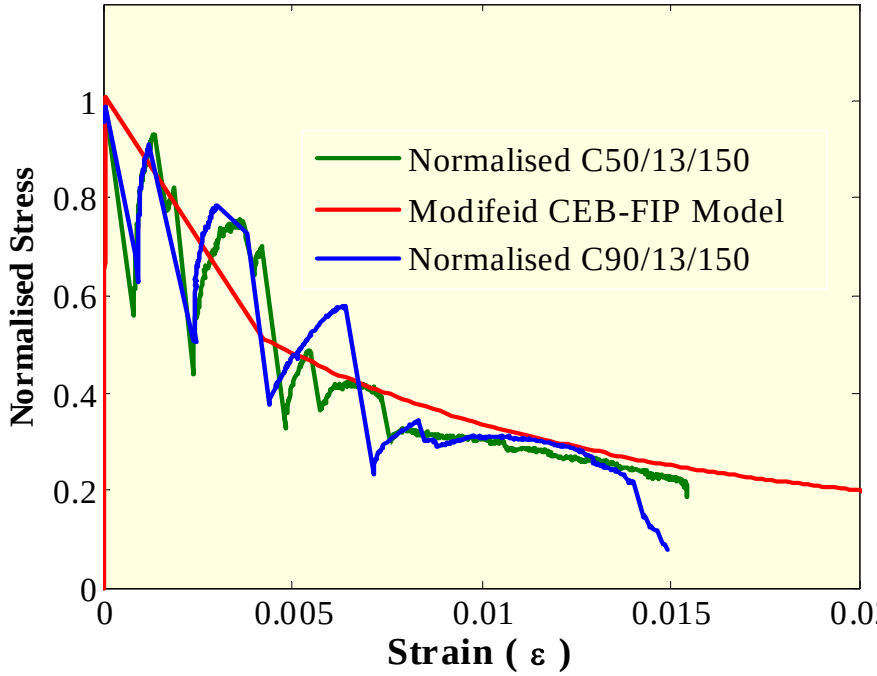
$$\varepsilon_m = \varepsilon_s \left[1 - 0.5 \left(\frac{f_{scr}}{f_f} \right)^2 \right]$$

Verification of the model for much wider data set

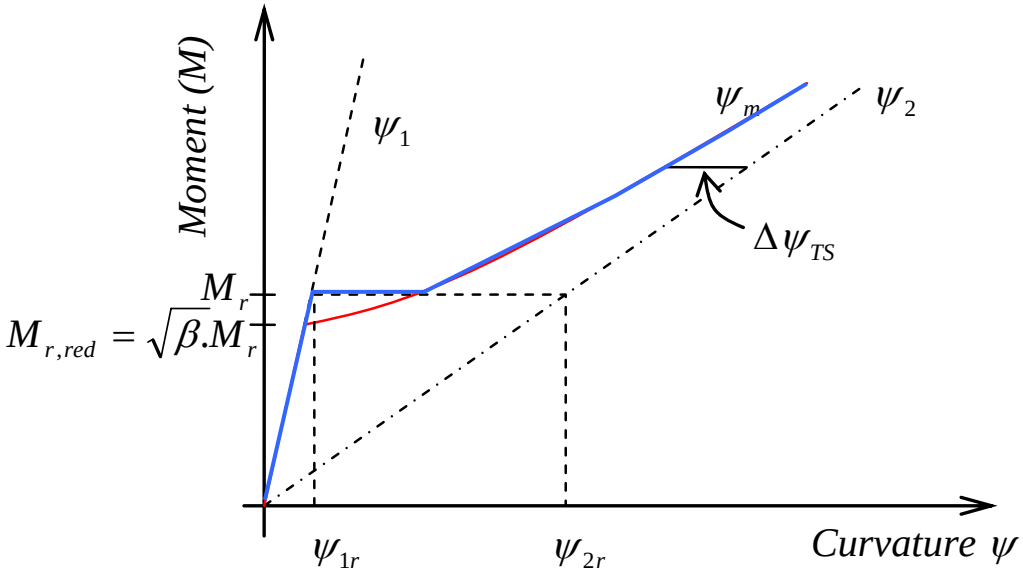








Incorporating Tension stiffening effect for deflection –Simplified approach



Moment curvature relationship

$$\psi_m = \psi_1$$

$$\psi_m = \psi_2 - \Delta\psi_{TS}$$

$$\Delta\psi_{TS} = (\psi_{2r} - \psi_{1r})\beta\frac{M_r}{M}$$

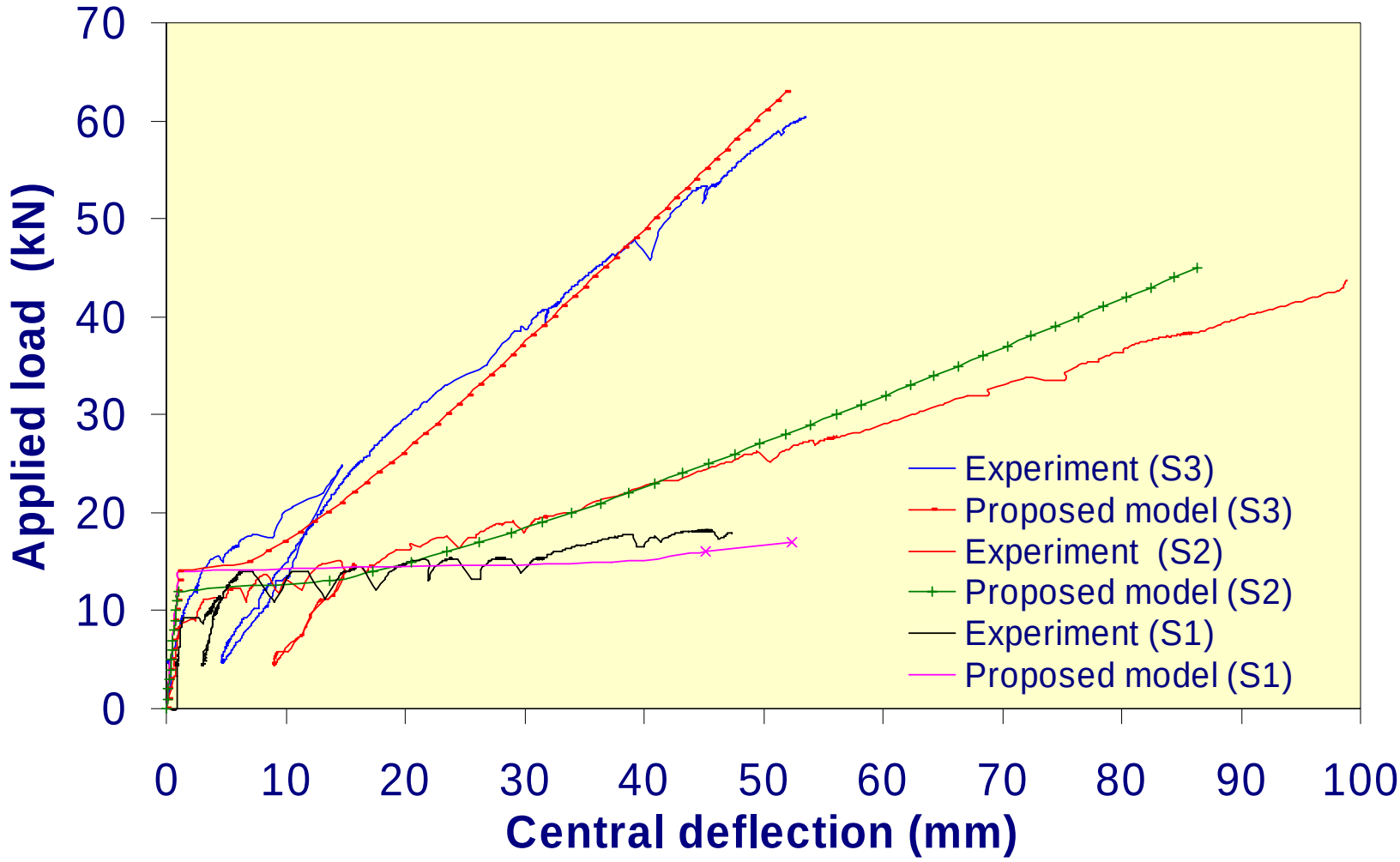
$$\psi_m = \psi_2 - (\psi_2 - \psi_1)\beta\left(\frac{M_r}{M}\right)^2$$

Simplified deflection relationship

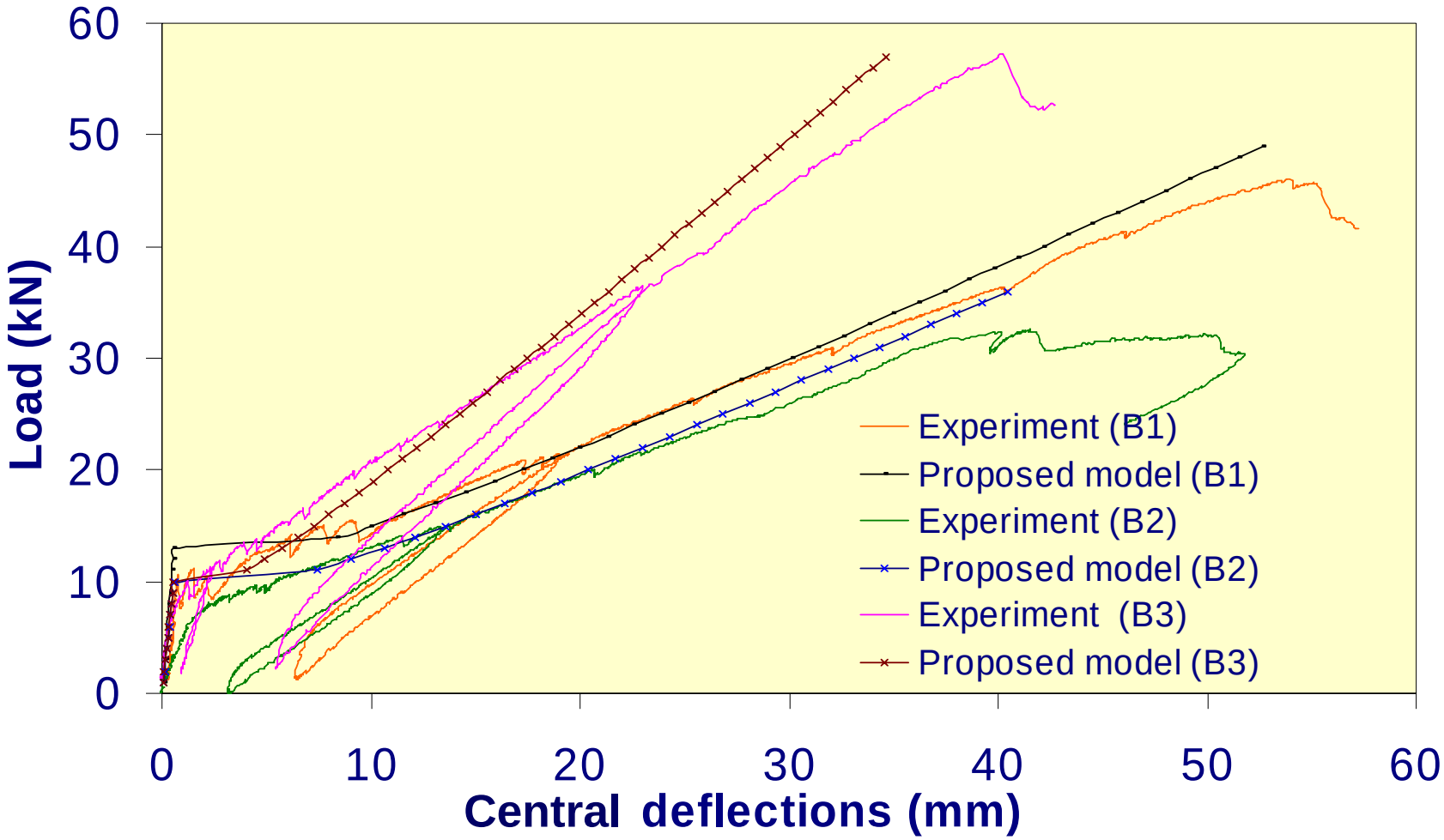
$$a = a_2 - (a_1 - a_2)\beta\left(\frac{M_{rD}}{M_D}\right)^2$$

$$a = a_2 - (a_1 - a_2)0.5\left(\frac{M_{rD}}{M_D}\right)^2$$

Comparing experimental results with Simplified approach- Slabs

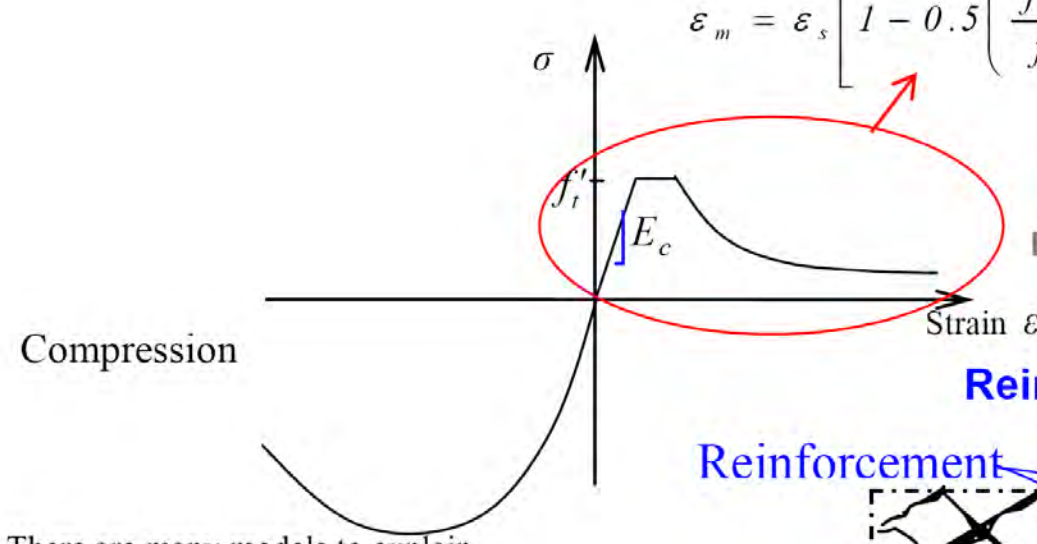


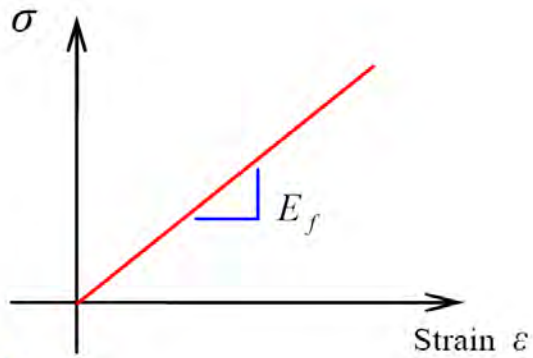
Comparing experimental results with Simplified approach-Beams



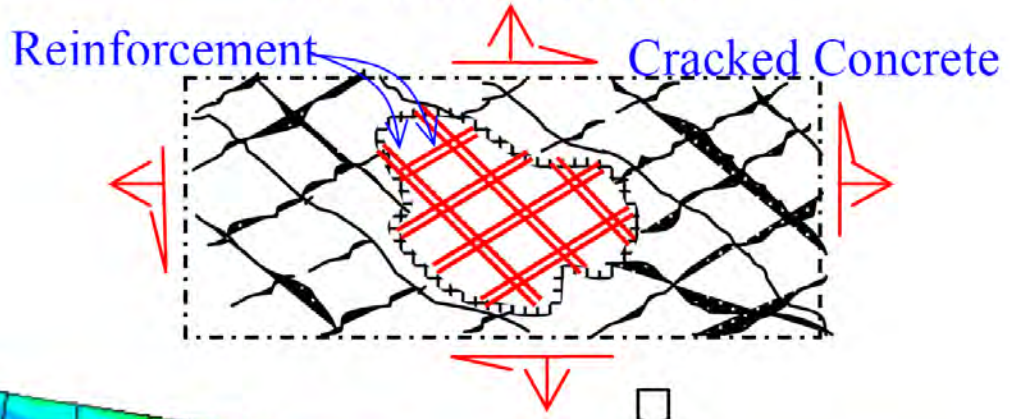
FEM analysis using tension stiffening model

$$\varepsilon_m = \varepsilon_s \left[1 - 0.5 \left(\frac{f_{scr}}{f_f} \right)^2 \right]$$

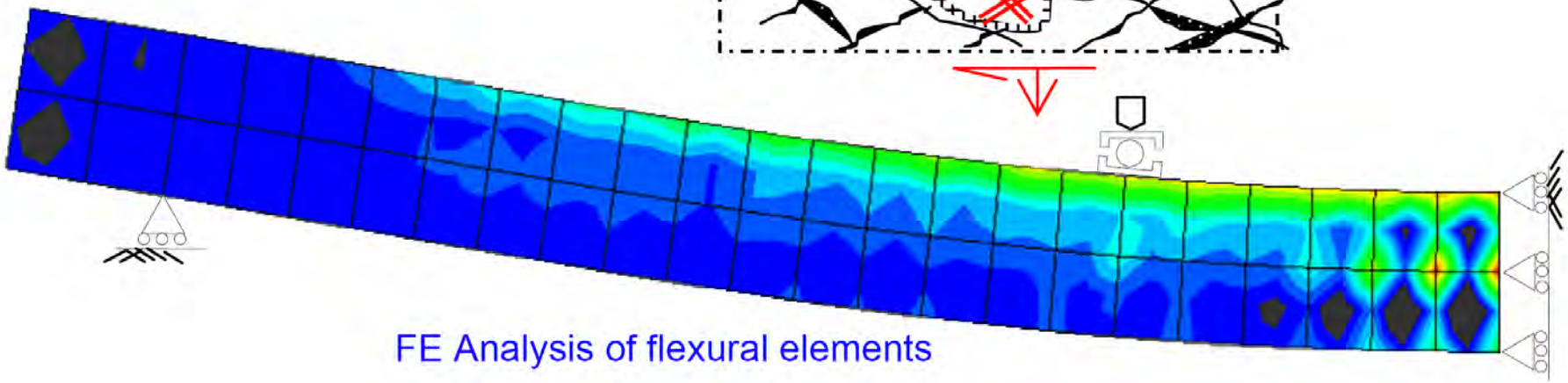




Reinforced concrete

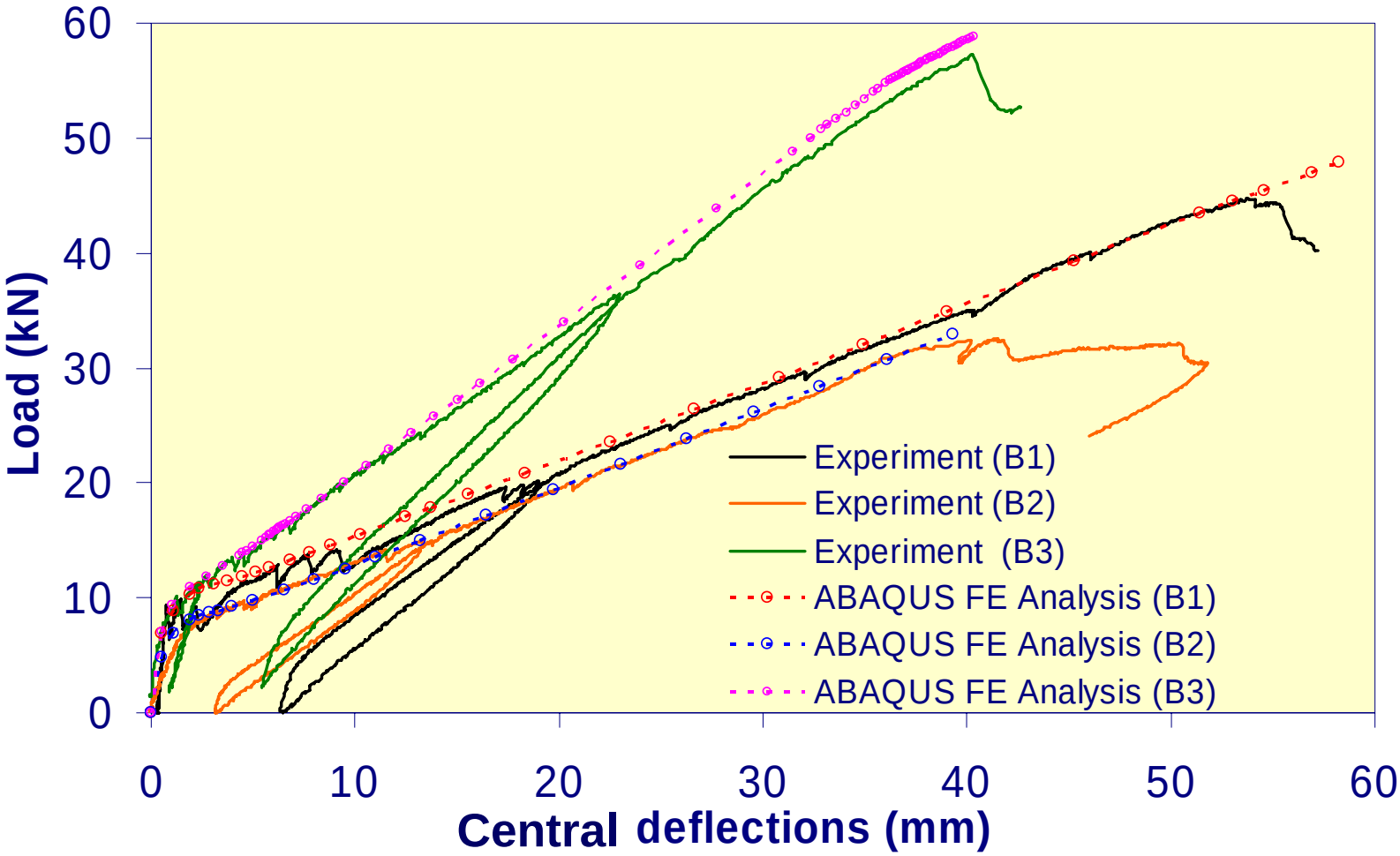


There are many models to explain the compression behaviour
BS8110 Part II, CEB-FIP

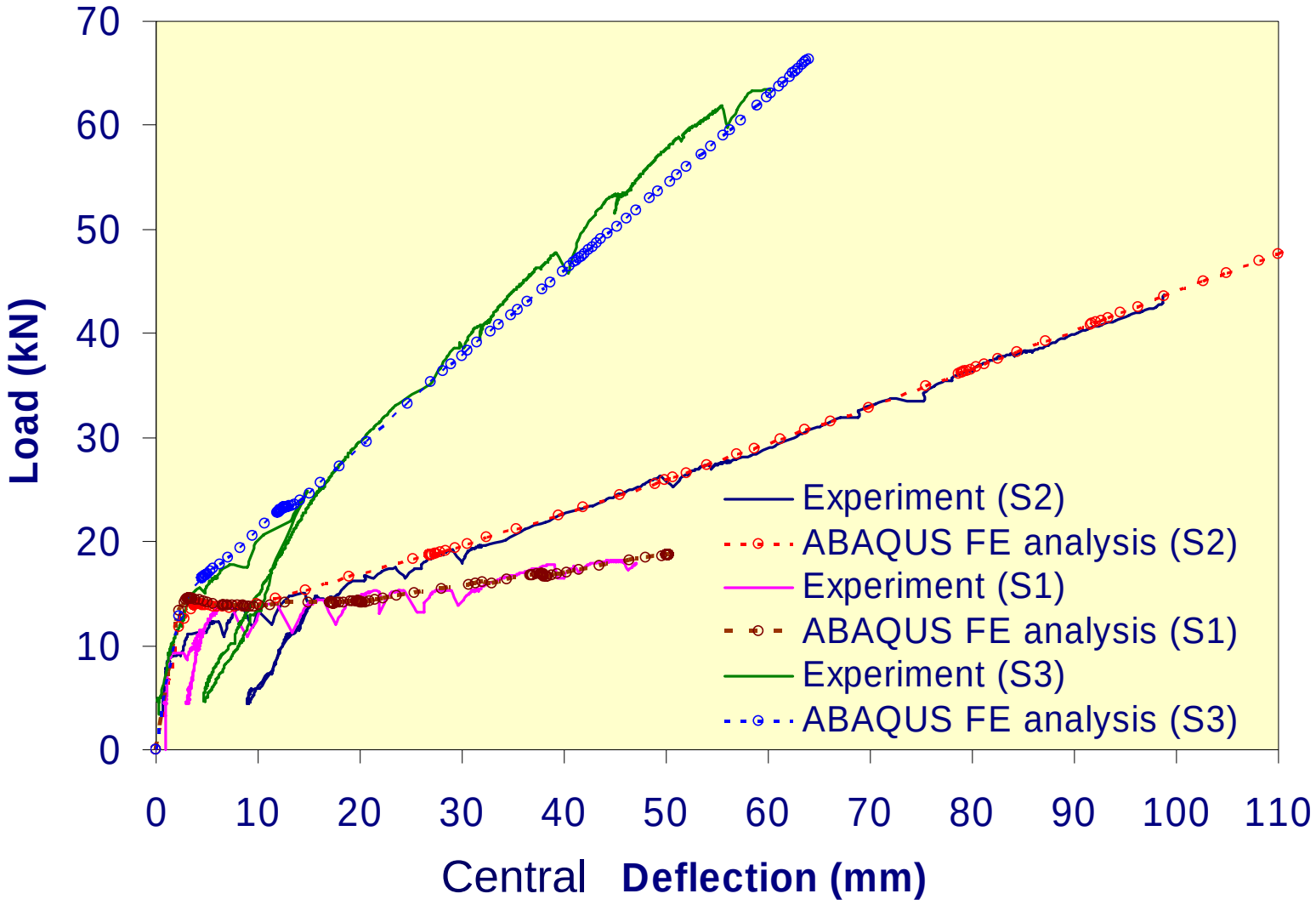


FE Analysis of flexural elements

Comparing Experimental results with FEA (Beams)



Comparing experimental results with FEA (Slabs)



Conclusions and Remarks

- Direct tension test is used to study the tension stiffening effect

- Existing ACI equation are not suitable for predicting tension stiffening effect or deflections

- Study proposes accurate model to account for tension stiffening effect

$$\varepsilon_{cf} = \varepsilon_f \left[1 - 0.5 \left(\frac{f_{scr}}{f_f} \right)^2 \right]$$

- With the proposed tension stiffening model accurate consistent deflection predictions is possible.

- Simplified version of deflection predictions are also shown possible with the proposed equation

$$a = a_2 - (a_1 - a_2)0.5 \left(\frac{M_{rD}}{M_D} \right)^2$$