MODELS FOR TENSION STIFFENING AND DEFLECTIONS OF GFRP-RC

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Outline

• Introduction

• Problem definition

• Methodology
  - Direct tension studies to quantify Tension stiffening effect

• Modelling tension behaviour GFRP-RC

• Modelling Deflections

• Conclusions and remarks
Introduction: GFRP RC in Construction

Corrosions of Steel reinforcements

Use of GFRP for bridge deck construction (Franklin county bridge Virginia)

GFRP bars

Steel bars

Stiffness of GFRP compared to Steel

Graph showing stress vs. strain for GFRP and steel bars.
In ACI

\[ \Delta = \frac{kPl^3}{EI_{eff}} \]

Branson’s equation for \( I_{eff} \)

\[ I_{eff} = I_g \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ I - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \]

ACI 440 approach

\[ I_{eff} = I_g \beta_d \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ I - \left( \frac{M_{cr}}{M_a} \right)^3 \right] \]

\[ \beta_d = \alpha_b \left[ \frac{E_f}{E_s} + 1 \right] \]

\[ \alpha_b = 0.5 \]

\[ \Delta_{error} = \text{Experimental Deflection minus the Deflection by Branson’s Equation, both at service level (50% ultimate load)} \]

(courtesy Toutanjie et al. (2003), Construction and Building Material)
### Test on deflections – beam series

<table>
<thead>
<tr>
<th>Rebar type</th>
<th>Beam type</th>
<th>Cover (mm)</th>
<th>Reinforcement</th>
<th>Rein. Ratio $\rho = \frac{A_s}{bh}$ (%)</th>
<th>$f_{cu}$ (MPa)</th>
<th>Failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>B1</td>
<td>31</td>
<td>1Ø12.7 mm</td>
<td>0.57</td>
<td>91</td>
<td>Bar failure</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>31</td>
<td>1Ø12.7 mm</td>
<td>0.57</td>
<td>46</td>
<td>Con. Crushing</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>31</td>
<td>2Ø12.7 mm</td>
<td>1.15</td>
<td>46</td>
<td>Con. Crushing</td>
</tr>
</tbody>
</table>

![Diagram of beam test setup](image-url)
Tests on deflections – beam series (B3)

$\rho = 1.15$

![Graph showing comparison of load vs. central deflections between experiment and ACI 440 models for beam series (B3)]
Tests on deflections – beam series (B2)

\[ \rho = 0.57 \]

- ACI 440 (B2)
- Experiment (B2)
Tests on deflections – beam series (B1)

\[ \rho = 0.57 \]

![Graph showing load vs. central deflections for Experiment (B1) and ACI 440 (B1)]
## Tests on deflections – Slab series

<table>
<thead>
<tr>
<th>Rebar type</th>
<th>Slab</th>
<th>Cover, (mm)</th>
<th>Reinforcement</th>
<th>Reinforcement Ratio ( \rho = \frac{A_s}{bh} (%) )</th>
<th>Concrete Cylinder strength “MPa”</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>S1</td>
<td>27.5</td>
<td>5∅6mm</td>
<td>0.24</td>
<td>43</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>31</td>
<td>5∅9.53mm</td>
<td>0.59</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>S3</td>
<td>40</td>
<td>5∅19.05mm</td>
<td>2.38</td>
<td>39</td>
</tr>
</tbody>
</table>

![Diagram of slab series with dimensions and transverse rebars]
Test on deflections – beam series (S3)

\[ \rho = 2.38 \]
\[ \rho = 0.59 \]
Test on deflections – beam series (S1)

\[ \rho = 0.24 \]
Tension Stiffening for FRP Design

- No general agreement on tension stiffening

\[ I_{\text{eff}} = I_g \left( \frac{M_{\text{cr}}}{M_a} \right)^3 + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^3 \right] \]

**ACI Branson’s**

\[ I_{\text{eff}} = I_g \beta_d \left( \frac{M_{\text{cr}}}{M_a} \right)^3 + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^3 \right] \]

**ACI 440**

\[ I_{\text{m}} = \frac{23 I_{\text{cr}} I_e}{8 I_{\text{cr}} + 15 I_e}, \quad I_{\text{c}} = \text{ACI} \quad I_{\text{eff}} \]

**Faza et. al B**

\[ I_{\text{eff}} = I_g \left( \frac{M_{\text{cr}}}{M_a} \right)^{5.5} + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^{5.5} \right] \]

**Alsayed et. al A**

\[ 1 < \frac{M_a}{M_{\text{cr}}} < 3 \Rightarrow I_{\text{eff}} = I_{\text{cr}} \left[ 1.40 - \frac{2}{15} \left( \frac{M_a}{M_{\text{cr}}} \right) \right] \quad \text{Alsayed et. al B} \]
Research Approach

• Study the tension stiffening effect at fundamental level

• Develop a suitable way to incorporate tension stiffening in deflection

Studying tension stiffening alone was important at the time as it is necessary for Modelling GFRP-RC using FE Method based on smeared crack approach.
Tension stiffening of concrete is defined as:

the ability of concrete to carry tension between cracks and provide extra stiffness for RC in tension.

- Serviceability often governs GFRP-RC design
- Tension stiffening is very important for the determination of deflections and crack widths at low load levels
Strain Softening Behaviour Concrete

Test results bar stress Vs overall strain

Average stress strain behaviour of concrete
## Parametric Study

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Concrete strength ($f'_c$)</th>
<th>Bar Diameter ($\phi$)</th>
<th>Dimension $b \times d \times l$</th>
<th>Reinforcement ratio ($\rho$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C50/13/100</td>
<td>46</td>
<td>12.7</td>
<td>100×100×1500</td>
<td>1.26</td>
</tr>
<tr>
<td>C50/13/150</td>
<td>46</td>
<td>12.7</td>
<td>150×150×1500</td>
<td>0.56</td>
</tr>
<tr>
<td>C50/13/200</td>
<td>46</td>
<td>12.7</td>
<td>200×200×1500</td>
<td>0.32</td>
</tr>
<tr>
<td>C90/13/100</td>
<td>91</td>
<td>12.7</td>
<td>100×100×1500</td>
<td>1.26</td>
</tr>
<tr>
<td>C90/13/150</td>
<td>91</td>
<td>12.7</td>
<td>150×150×1500</td>
<td>0.56</td>
</tr>
<tr>
<td>C50/19/150</td>
<td>46</td>
<td>19.1</td>
<td>150×150×1500</td>
<td>1.27</td>
</tr>
<tr>
<td>C50/19/200</td>
<td>46</td>
<td>19.1</td>
<td>200×200×1300</td>
<td>0.72</td>
</tr>
<tr>
<td>C90/19/150</td>
<td>46</td>
<td>19.1</td>
<td>150×150×1300</td>
<td>1.27</td>
</tr>
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</tr>
</tbody>
</table>
Prediction of tension stiffening effect - Code based approach

Reduced cross sectional area

Original

$$A_e = \left( \frac{P_{cr}}{P_a} \right)^3 A_g + \left[ 1 - \left( \frac{P_{cr}}{P_a} \right)^3 \right] A_{cr}$$

Modified to account for weak FRP bond

$$A_e = \left( \frac{P_{cr}}{P_a} \right)^3 \beta_d A_g + \left[ 1 - \left( \frac{P_{cr}}{P_a} \right)^3 \right] A_{cr}$$

Composite strain for given bar strain

ACI

$$\varepsilon_m = \varepsilon_s \left[ 1 - K \left( \frac{f_{scr}}{f_{t}} \right)^2 \right]$$

CEB

$$f_{scr} = \frac{P_{cr}}{A_t} = f_t \left( \frac{1}{p} - 1 + n_f \right)$$
Model for Tension Stiffening effect of GFRP-RC

\[
\varepsilon_m = \varepsilon_s \left[ 1 - 0.5 \left( \frac{f_{scr}}{f_f} \right)^2 \right]
\]
Verification of the model for much wider data set
Incorporating Tension stiffening effect for deflection – Simplified approach

Moment curvature relationship

\[ \psi_m = \psi_1 \]
\[ \psi_m = \psi_2 - \Delta \psi_{TS} \]
\[ \Delta \psi_{TS} = (\psi_{2r} - \psi_{1r}) \beta \frac{M_r}{M} \]
\[ \psi_m = \psi_2 - (\psi_2 - \psi_1) \beta \left( \frac{M_r}{M} \right)^2 \]

Simplified deflection relationship

\[ a = a_2 - (a_1 - a_2) \beta \left( \frac{M_{rD}}{M_D} \right)^2 \]
\[ a = a_2 - (a_1 - a_2) 0.5 \left( \frac{M_{rD}}{M_D} \right)^2 \]
Comparing experimental results with Simplified approach- Slabs
Comparing experimental results with Simplified approach-Beams

Central deflections (mm)

Load (kN)

- Experiment (B1)
- Proposed model (B1)
- Experiment (B2)
- Proposed model (B2)
- Experiment (B3)
- Proposed model (B3)
FEM analysis using tension stiffening model

\[ \varepsilon_w = \varepsilon_s \left[ 1 - 0.5 \left( \frac{f_{scr}}{f_f} \right)^2 \right] \]

Compression

There are many models to explain the compression behaviour
BS8110 Part II, CEB-FIP

Reinforced concrete

Reinforcement

Cracked concrete

FE Analysis of flexural elements
Comparing Experimental results with FEA (Beams)

- Central deflections (mm)
- Load (kN)

- Experiment (B1)
- Experiment (B2)
- Experiment (B3)
- ABAQUS FE Analysis (B1)
- ABAQUS FE Analysis (B2)
- ABAQUS FE Analysis (B3)
Comparing experimental results with FEA (Slabs)
Conclusions and Remarks

• Direct tension test is used to study the tension stiffening effect

• Existing ACI equation are not suitable for predicting tension stiffening effect or deflections

• Study proposes accurate model to account for tension stiffening effect

\[ \varepsilon_{cf} = \varepsilon_f \left[ 1 - 0.5 \left( \frac{f_{scr}}{f_f} \right)^2 \right] \]

• With the proposed tension stiffening model accurate consistent deflection predictions is possible.

• Simplified version of deflection predictions are also shown possible with the proposed equation

\[ a = a_2 - (a_1 - a_2)0.5 \left( \frac{M_{rd}}{M_D} \right)^2 \]