

# TENSION STIFFENING BEHAVIOUR OF GFRP-RC

**Kypros Pilakoutas**  
**Harsha Sooriyaarachchi**  
**Ewan Byars**

***Centre for Cement and Concrete***

*Department of Civil and Structural Engineering, University of Sheffield,  
Sheffield, United Kingdom*

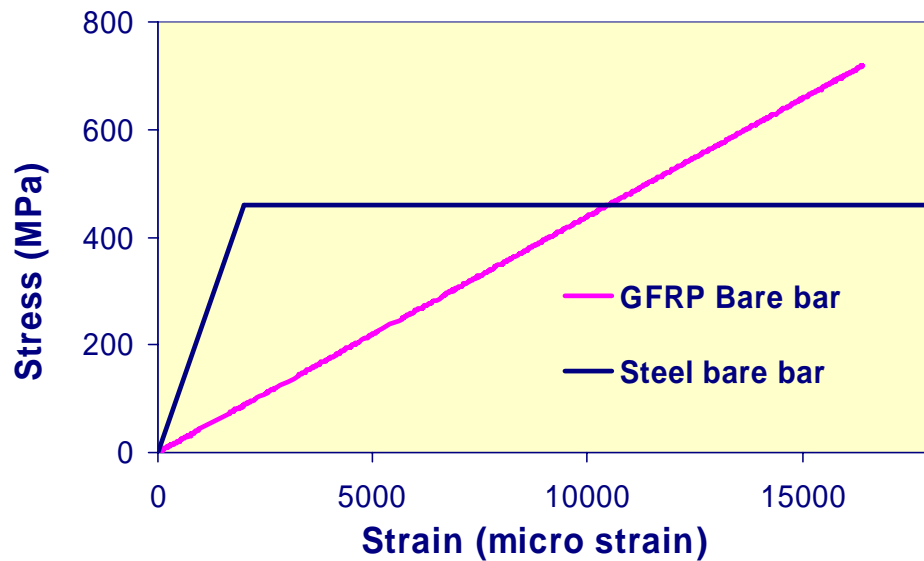
- **Importance of tension stiffening**
- **Tension stiffening in codes**  
-Deflection calculations
- **Direct tension studies**
- **Parametric studies**
- **Modelling**
- **Conclusions**



**GFRP bars**



**Use of GFRP for bridge deck construction**  
(Franklin county bridge Virginia)

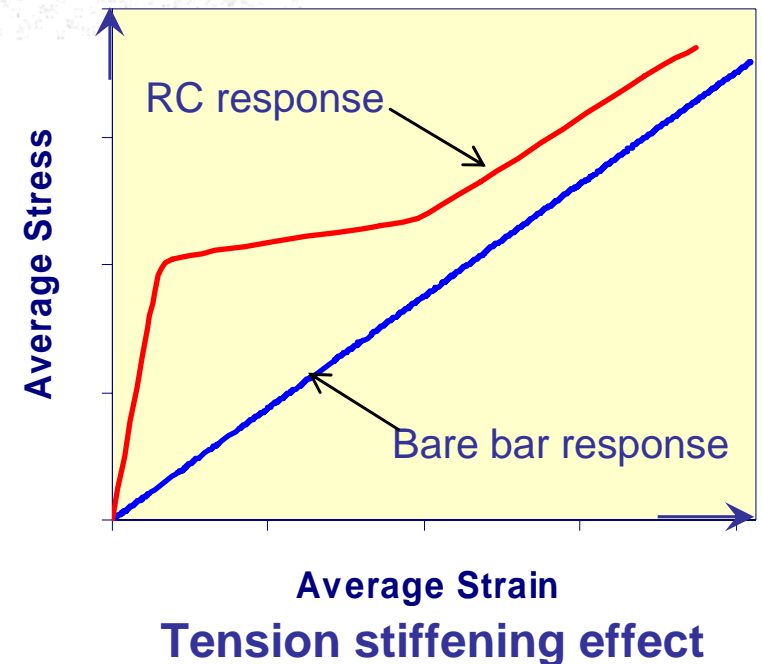
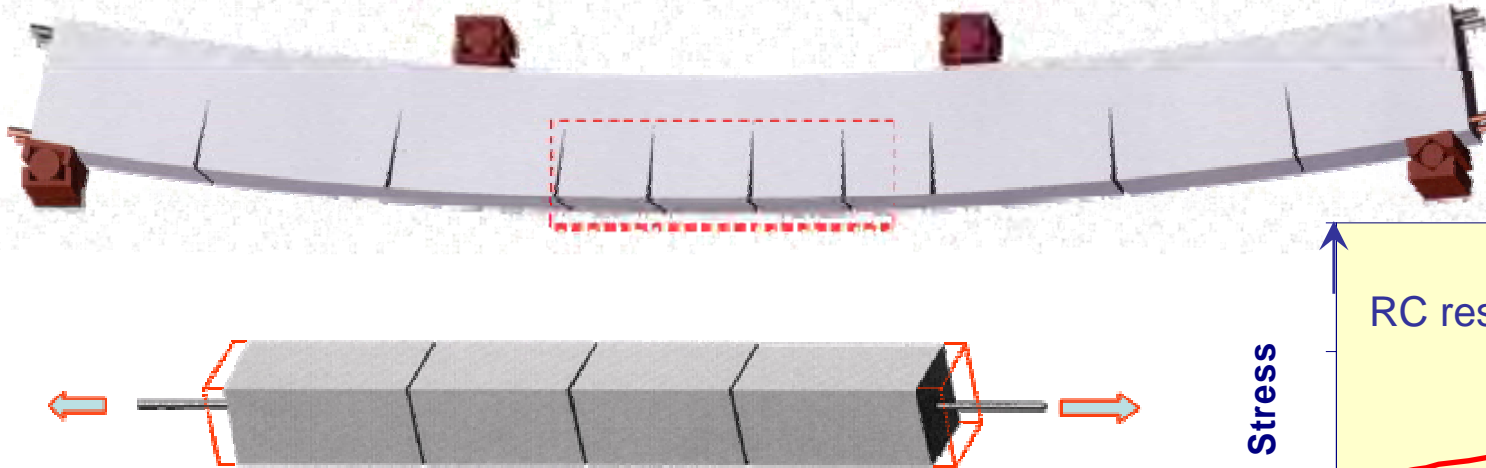


**Stiffness of GFRP compared to Steel**

Tension stiffening of concrete is defined as:

*the ability of concrete to carry tension between cracks and provide extra stiffness for RC in tension.*

- Serviceability often governs GFRP-RC design
- Tension stiffening is very important for the determination of deflections and crack widths at low load levels



In ACI

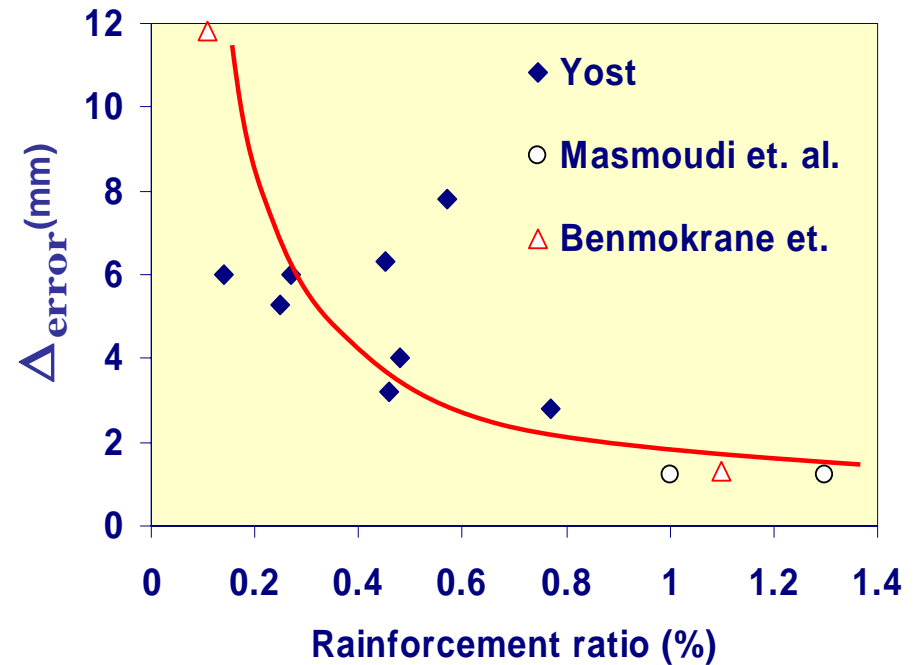
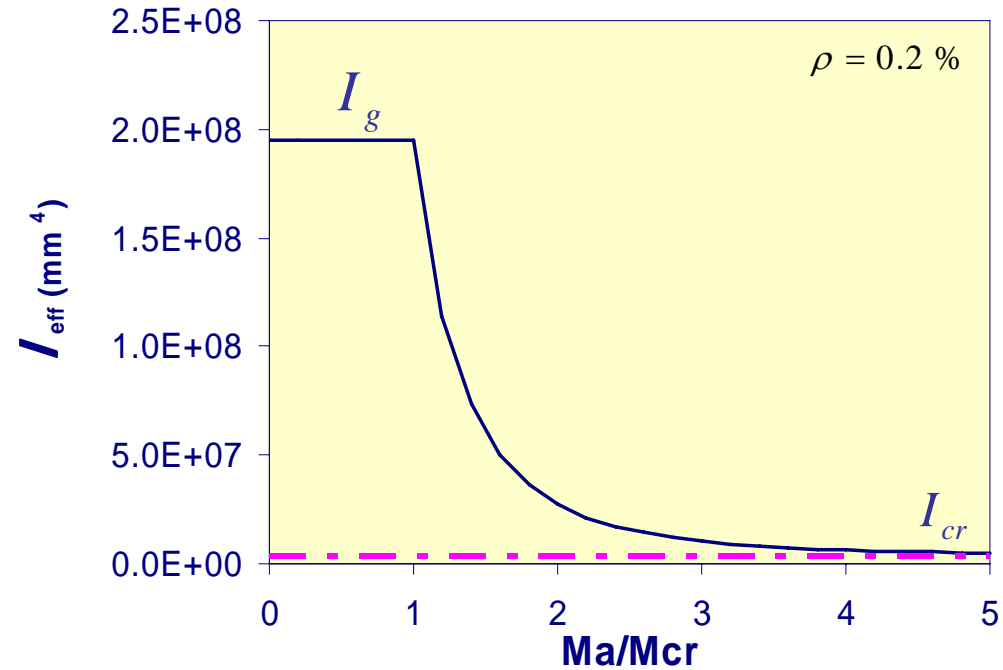
$$\Delta = \frac{kPl^3}{EI_{eff}}$$

Branson's equation for  $I_{eff}$

$$I_{eff} = I_g \left( \frac{M_{cr}}{M_a} \right)^3 + I_{cr} \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right]$$

$\Delta_{error}$  = Experimental Deflection minus the Deflection by Branson's Equation, both at service level (50% ultimate load)

(Toutanji et al. (2003), *Construction and Building Material*)



- No general agreement on tension stiffening

$$I_{\text{eff}} = I_g \left( \frac{M_{\text{cr}}}{M_a} \right)^3 + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^3 \right] \quad \text{ACI Branson's}$$

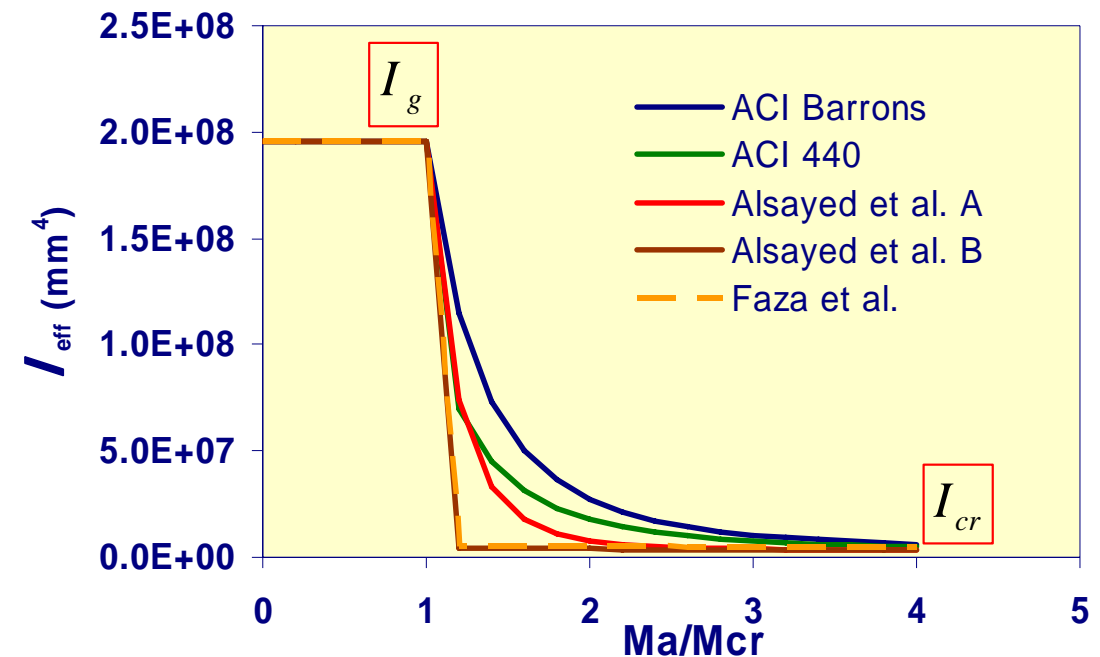
$$I_{\text{eff}} = I_g \beta_d \left( \frac{M_{\text{cr}}}{M_a} \right)^3 + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^3 \right] \quad \text{ACI 440}$$

$$I_m = \frac{23I_{\text{cr}}I_e}{8I_{\text{cr}} + 15I_e}, \quad I_e = \text{ACI } I_{\text{eff}} \quad \text{Faza et. al B}$$

$$I_{\text{eff}} = I_g \left( \frac{M_{\text{cr}}}{M_a} \right)^{5.5} + I_{\text{cr}} \left[ 1 - \left( \frac{M_{\text{cr}}}{M_a} \right)^{5.5} \right] \quad \text{Alsayed et. al A}$$

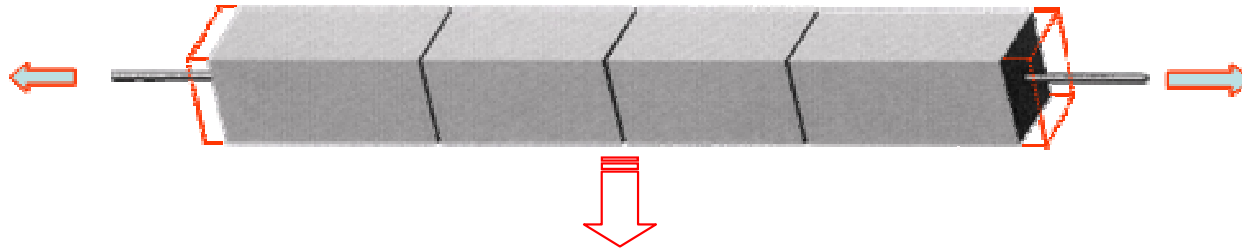
$$1 < \frac{M_a}{M_{\text{cr}}} < 3 \Rightarrow I_{\text{eff}} = I_{\text{cr}} \left[ 1.40 - \frac{2}{15} \left( \frac{M_a}{M_{\text{cr}}} \right) \right] \quad \text{Alsayed et. al B}$$

$$I_{\text{eff}} = I_{\text{cr}}$$

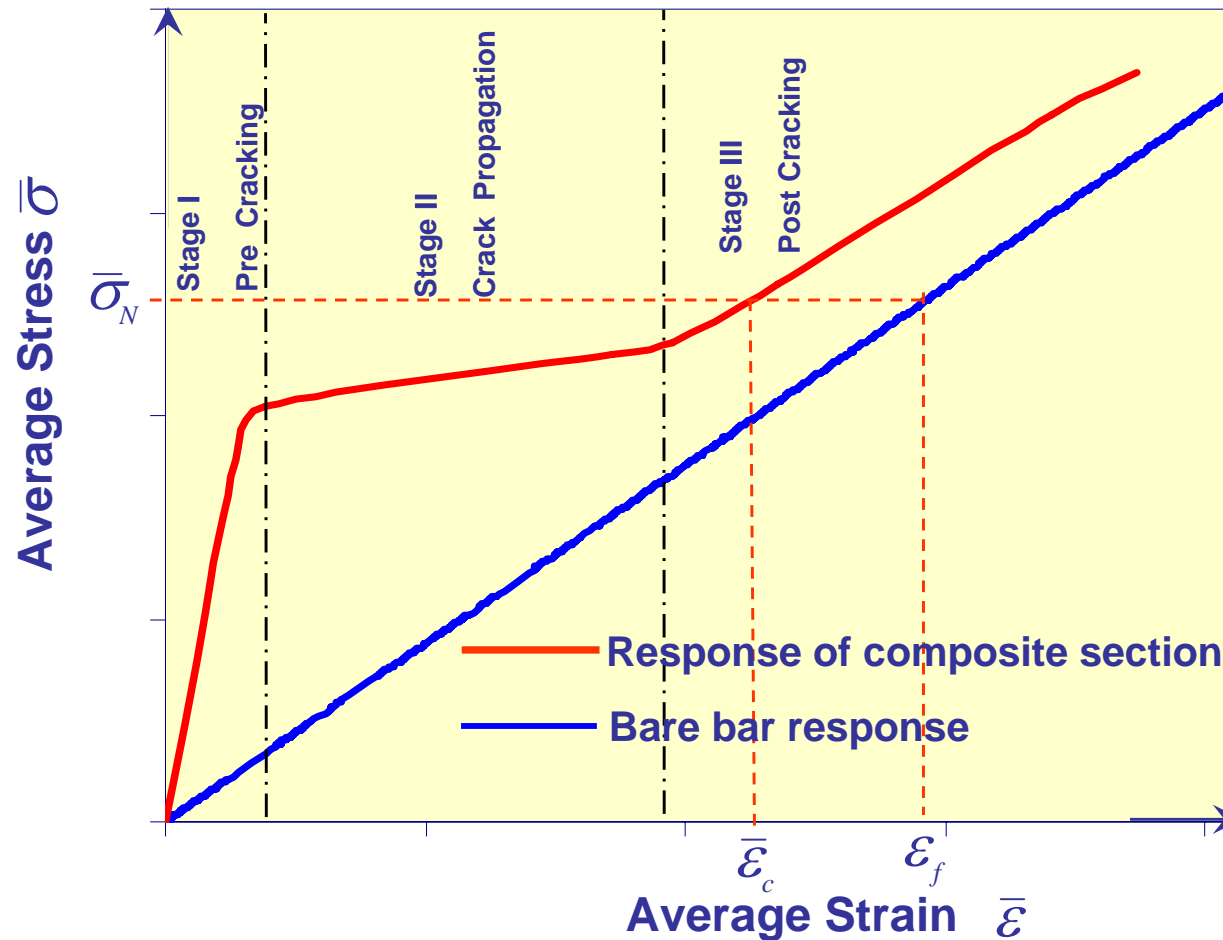


- Need to develop tension stiffening models for numerical analysis

## Reinforced concrete in tension



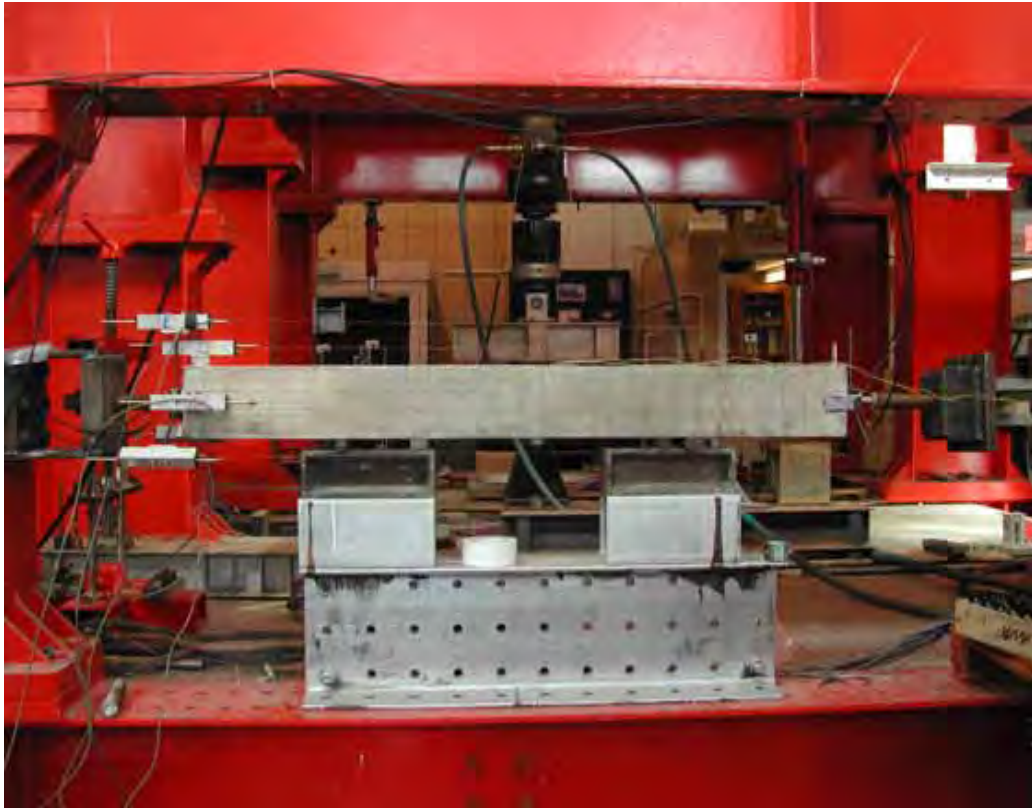
## Typical Test Results



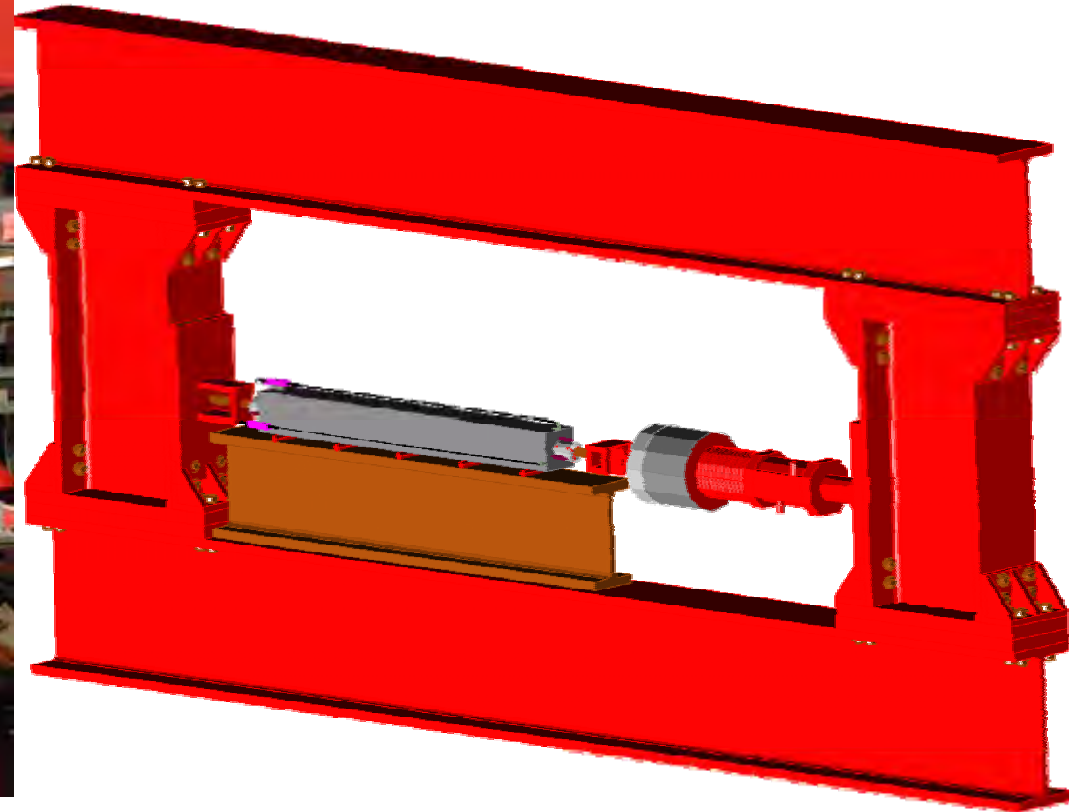
$\bar{\epsilon}_c$  Strain of the composite specimen at  $\bar{\sigma}_N$

$\bar{\epsilon}_f$  Strain of the bar at crack  $\bar{\sigma}_N$

## Testing Rig

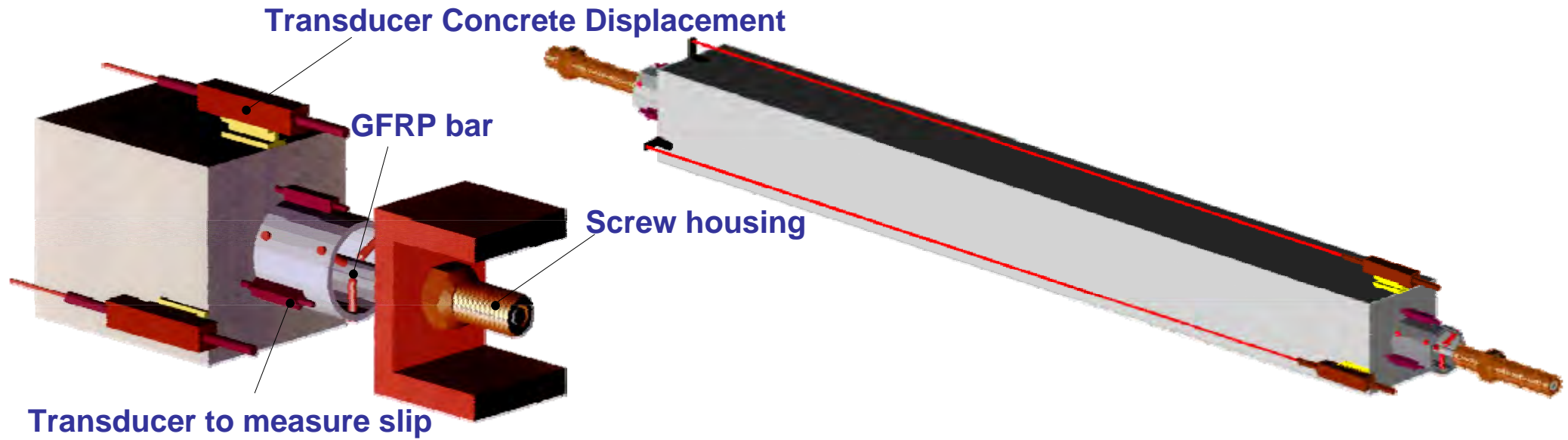


Photograph of the testing rig

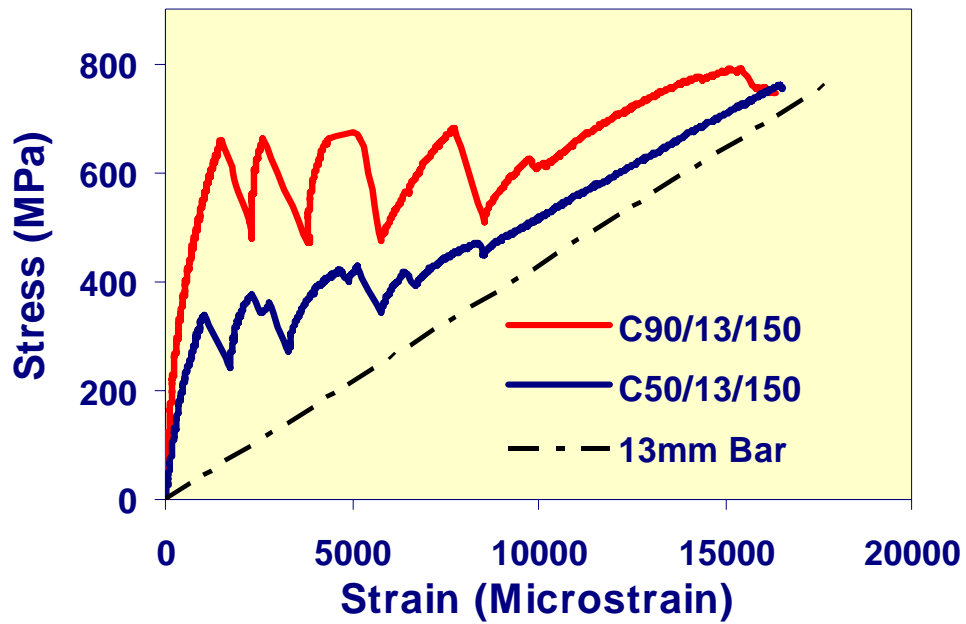


Schematic representation of the rig

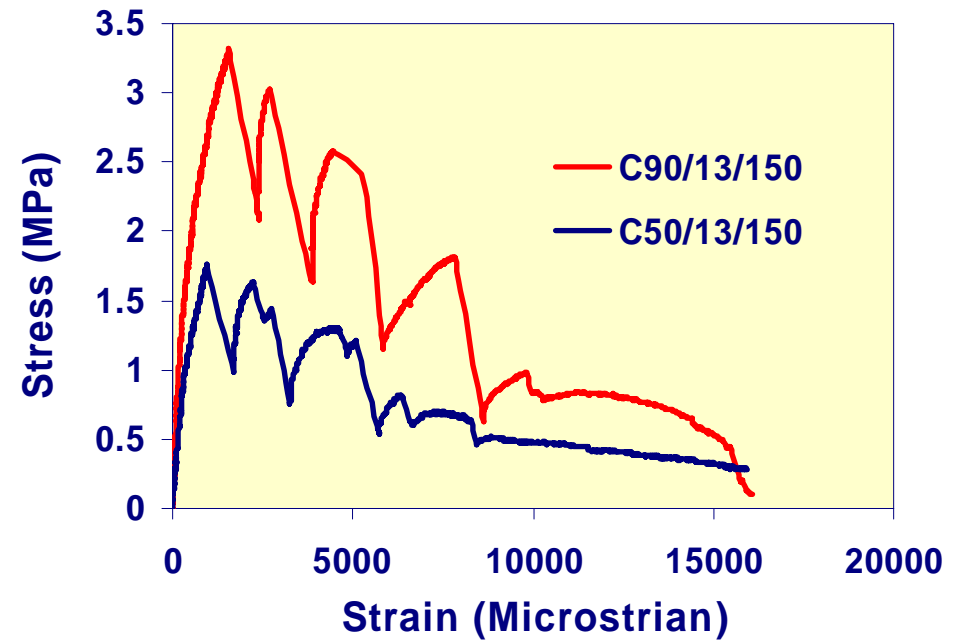
## Measuring Arrangement



**Experimental Set-up**

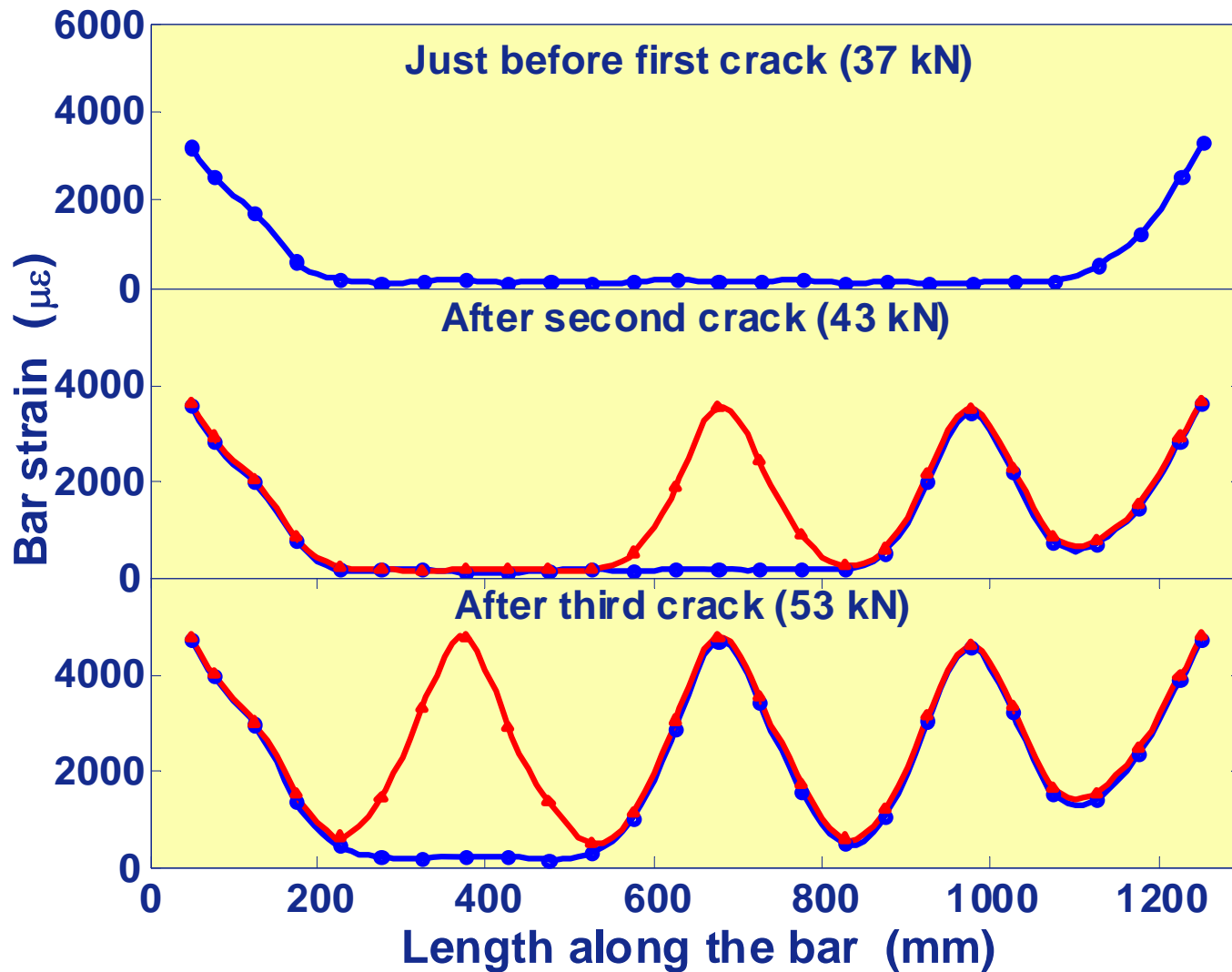
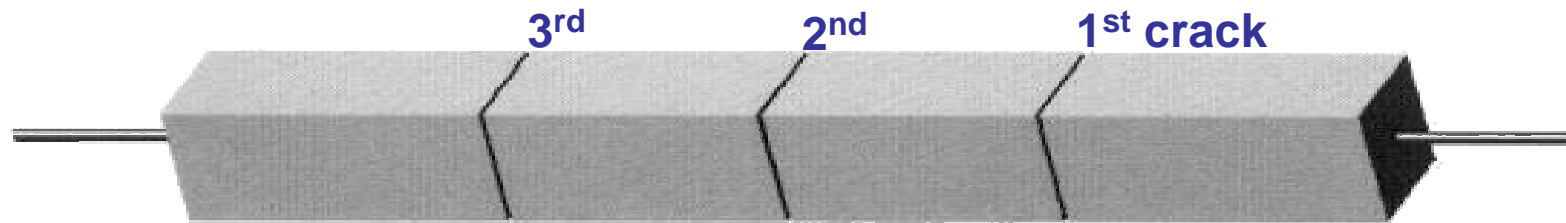


Test results bar stress Vs overall strain



Average stress strain behaviour of concrete

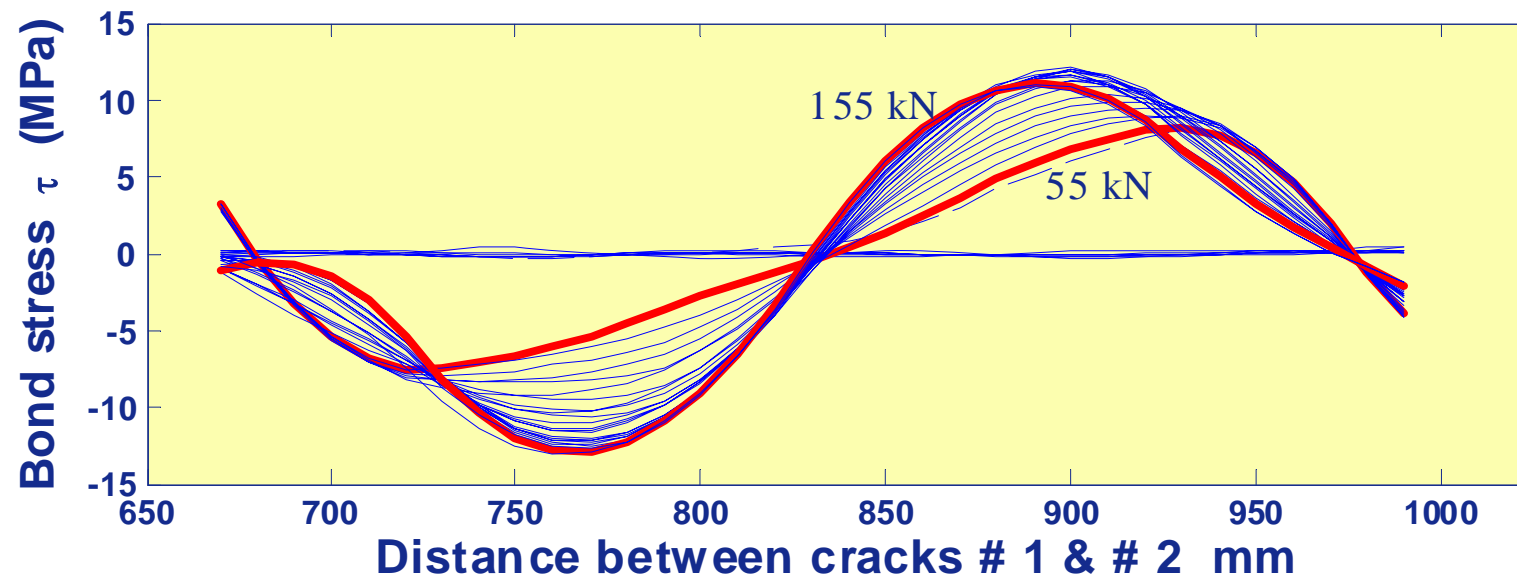
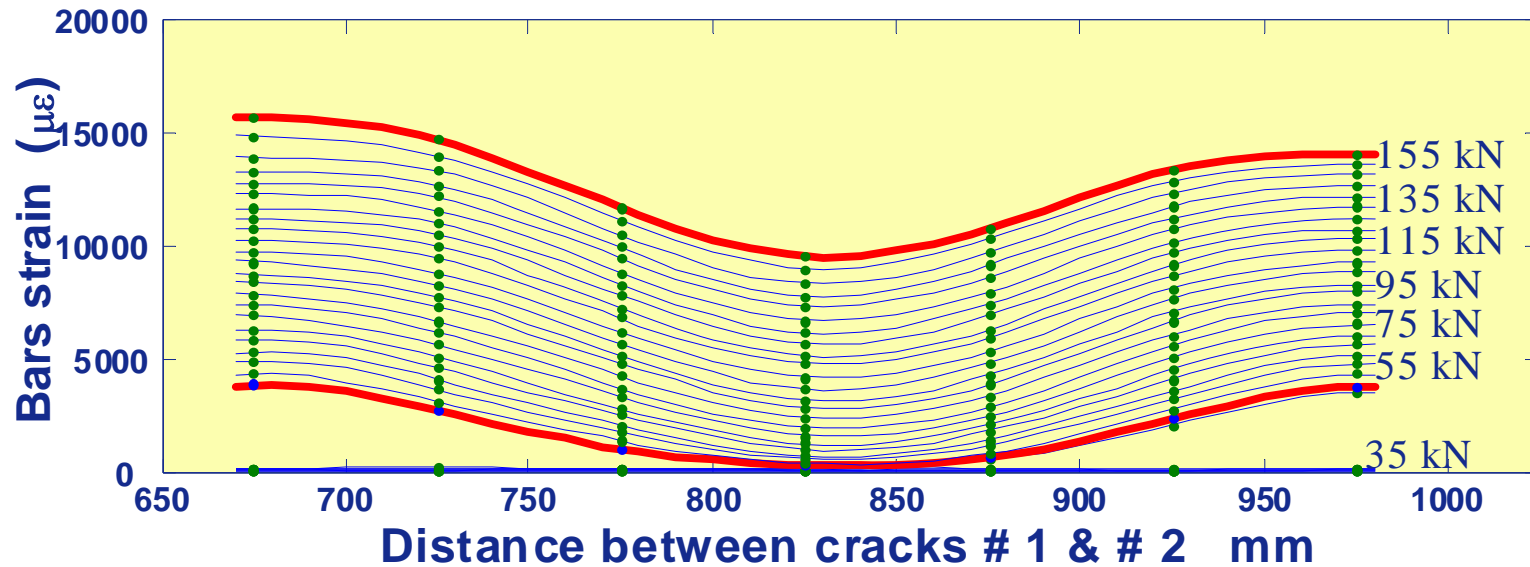
# Strain Distribution during Cracking



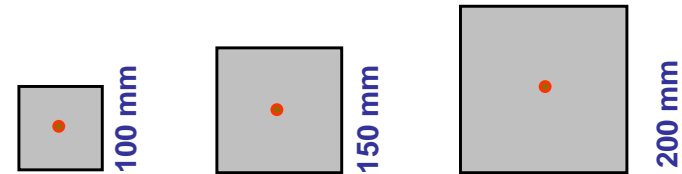
Internally strain gauged bar



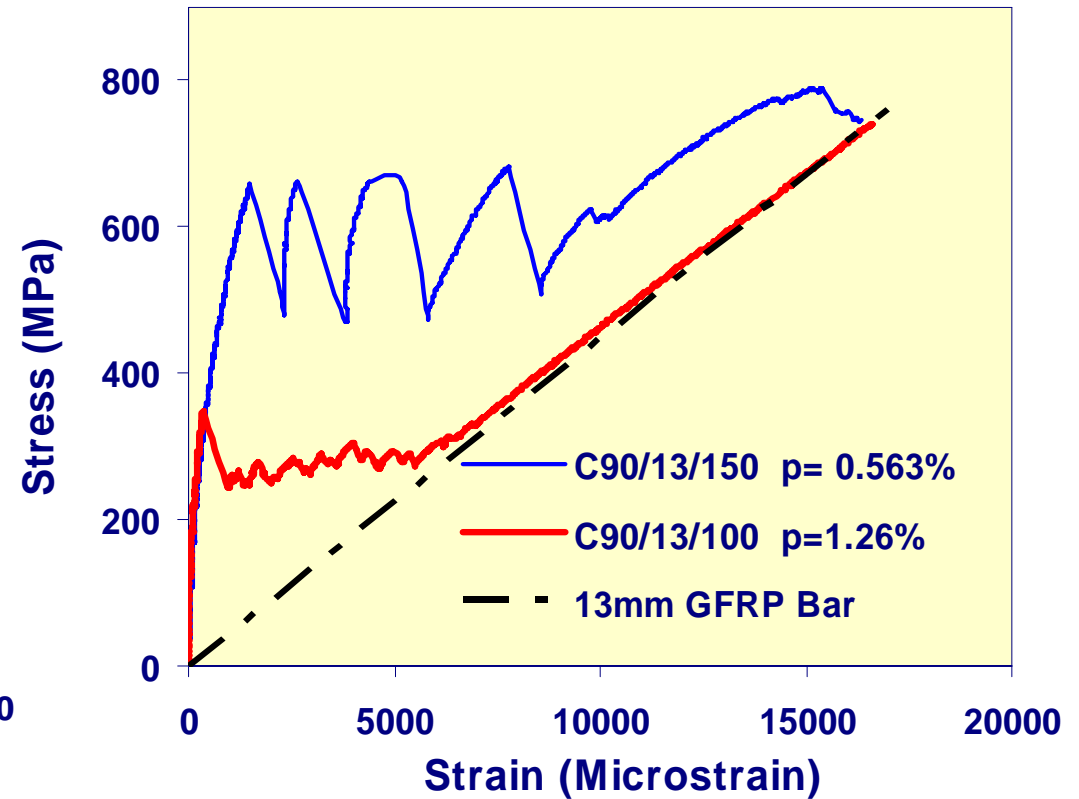
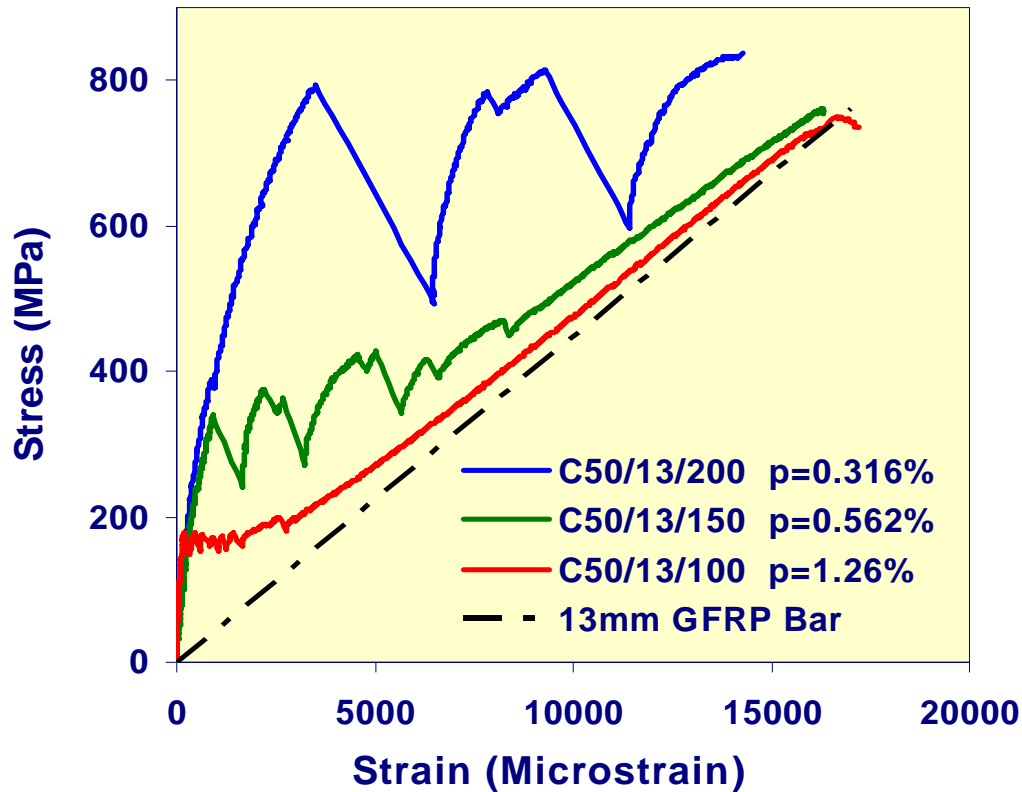
# Post Cracking Strain Profile



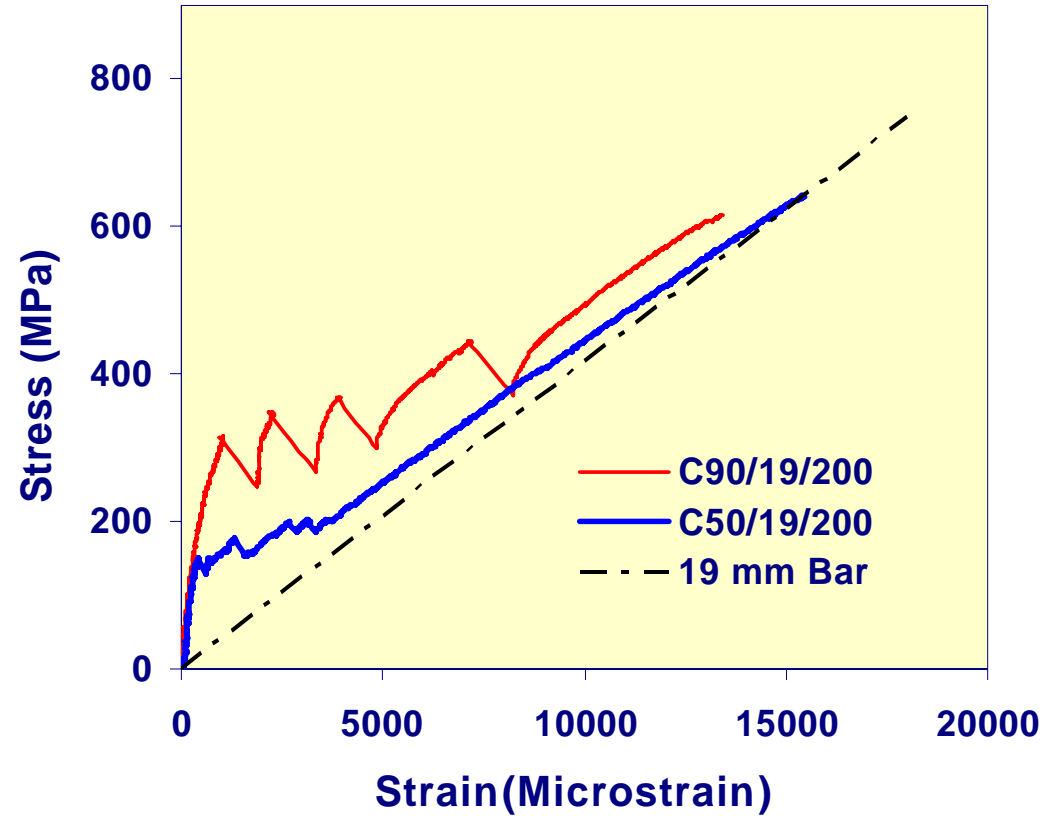
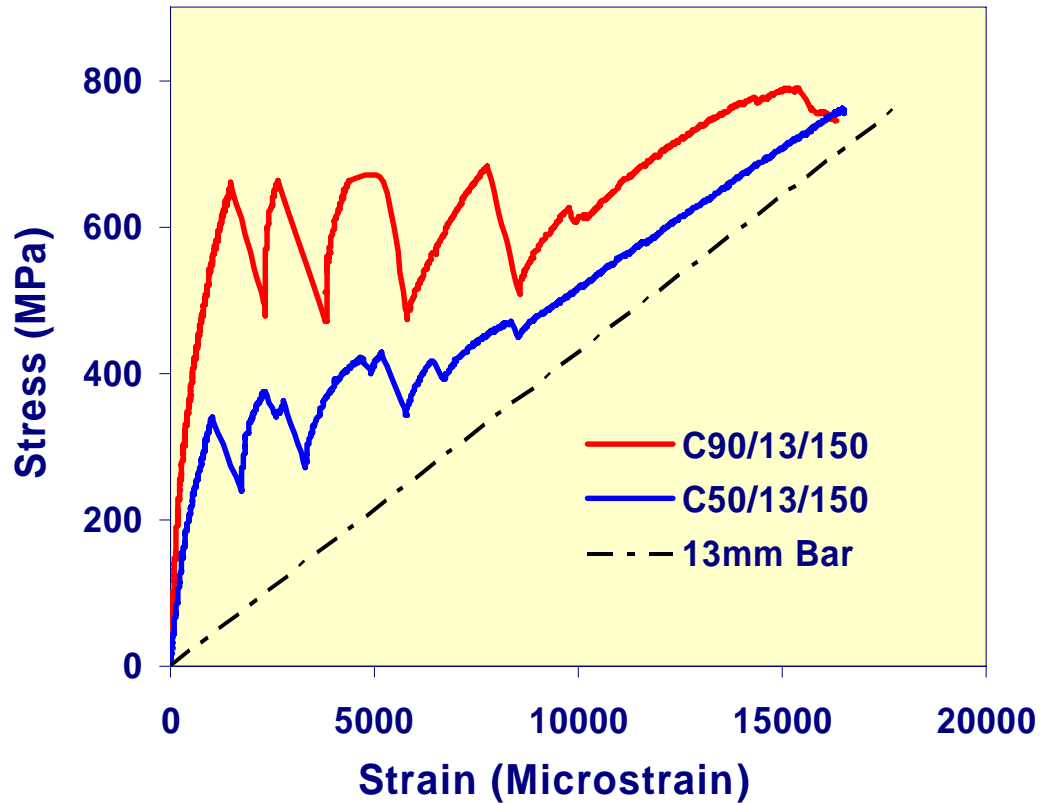
Typical strain and bond stress distribution between cracks



Specimen	Concrete strength ( $f_c'$ )	Bar Diameter ( $\phi$ )	Dimension $b \times d \times l$	Reinforcement ratio ( $\rho$ )
C50/13/100	52	12.7	100×100×1500	1.26
C50/13/150	52	12.7	150×150×1500	0.56
C50/13/200	52	12.7	200×200×1500	0.32
C90/13/100	91	12.7	100×100×1500	1.26
C90/13/150	91	12.7	150×150×1500	0.56
C50/19/150	52	19.1	150×150×1500	1.27
C50/19/200	52	19.1	200×200×1300	0.72
C90/19/150	52	19.1	150×150×1300	1.27
C90/19/200	91	19.1	200×200×1300	0.72
C50/19/200N	91	19.1	200×200×1300	0.72

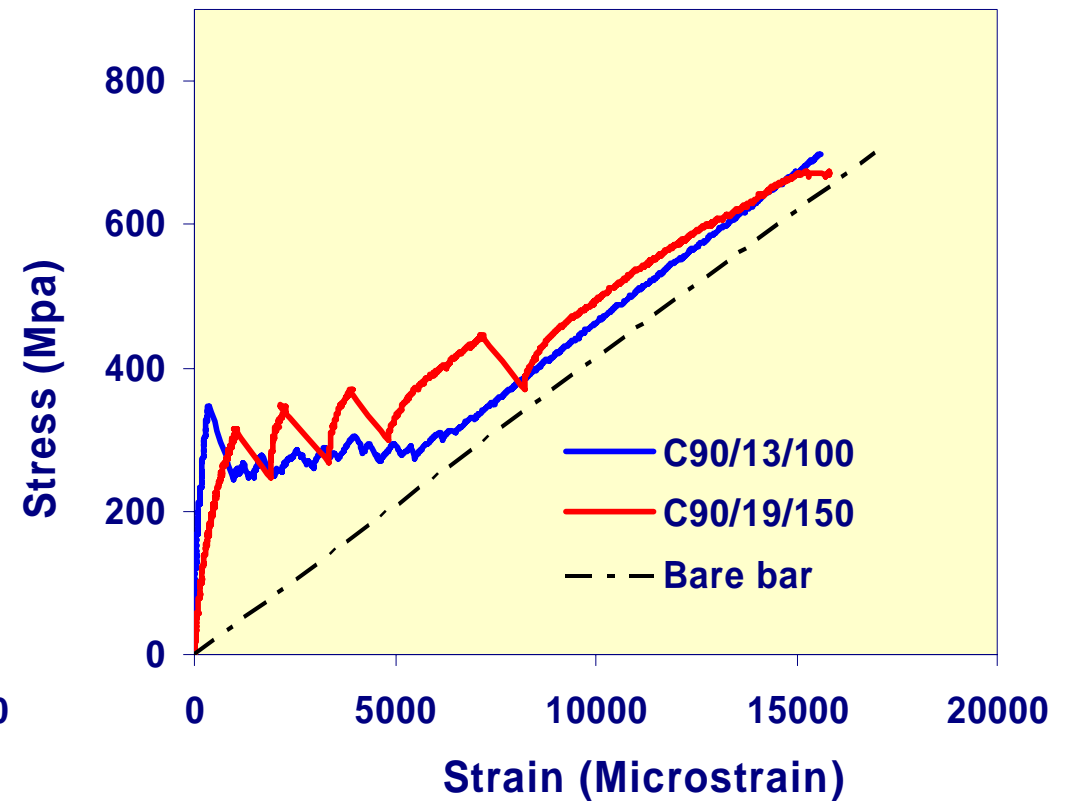
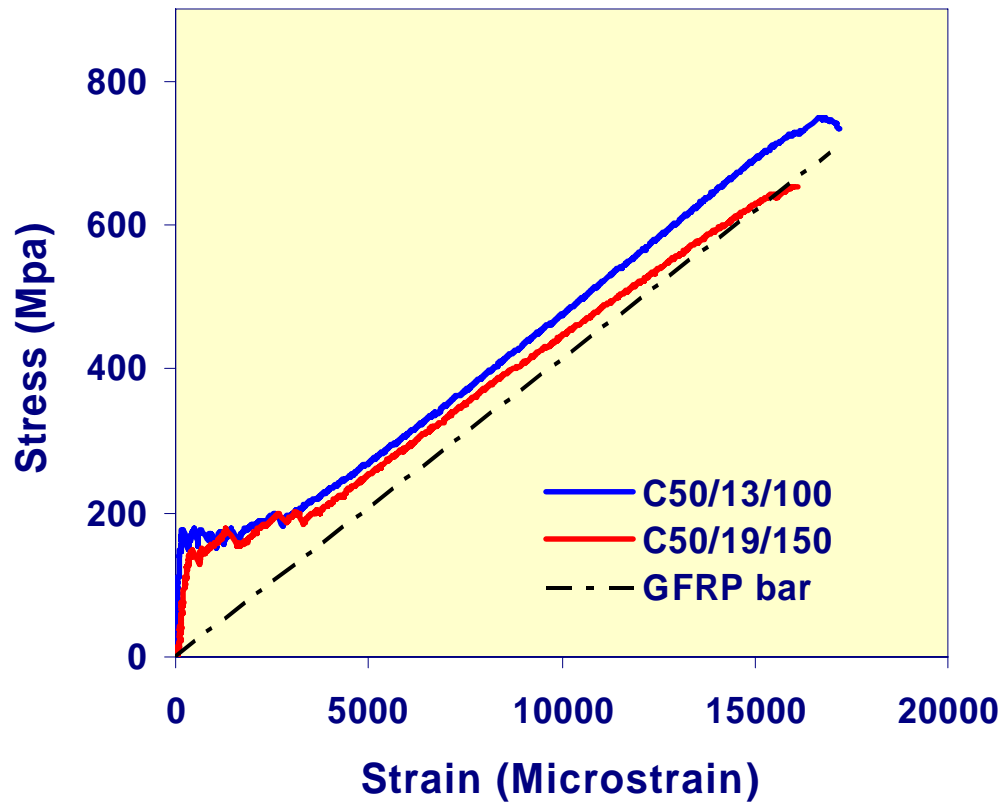


- Direct influence on tension stiffening effect

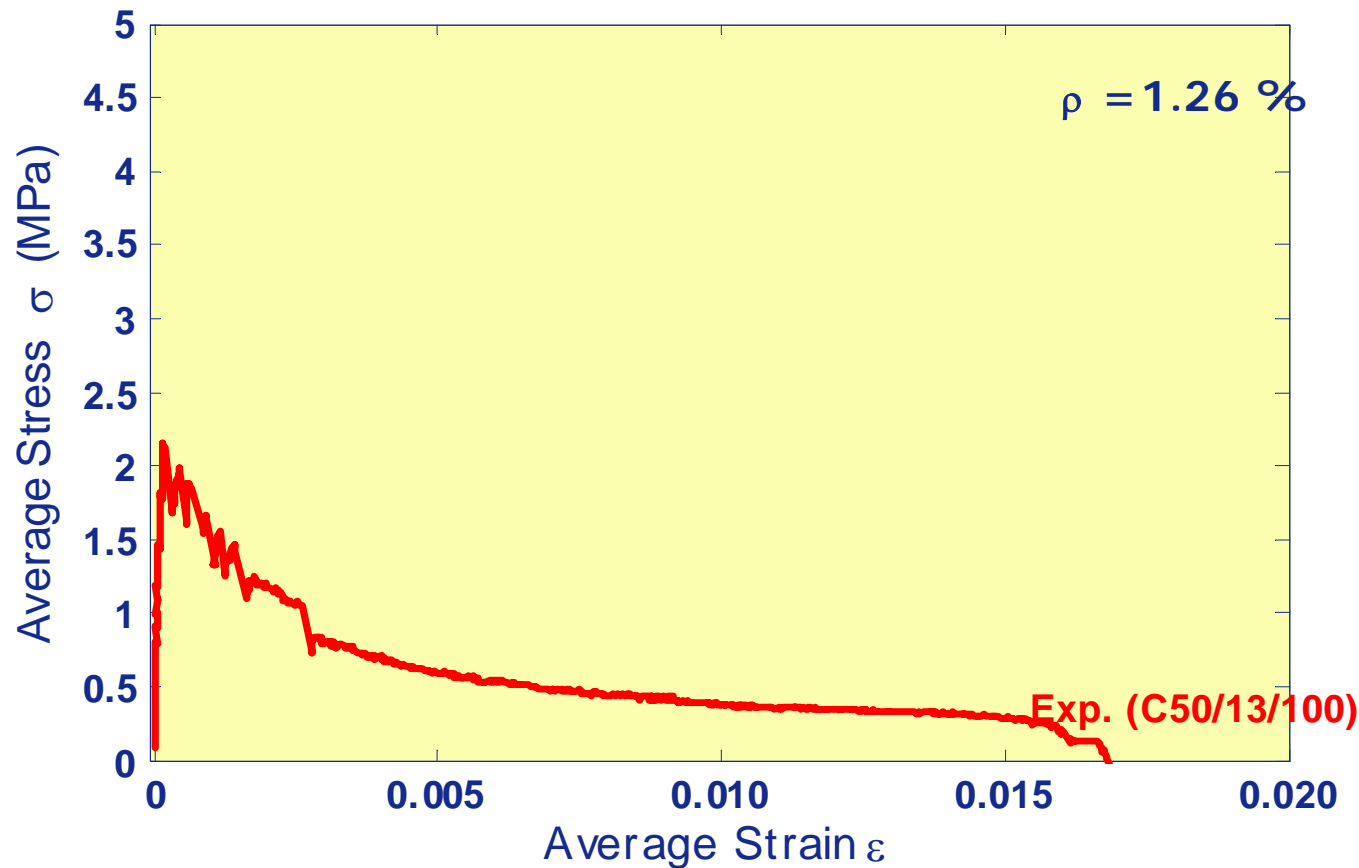


- Direct influence on tension stiffening effect

# Bar Diameter



- Bar diameter when compared at same reinforcement ratio has no significant influence of tension stiffening effect



## Reduced cross sectional area

Original 
$$A_e = \left[ \frac{P_{cr}}{P_a} \right]^3 A_g + \left[ 1 - \left( \frac{P_{cr}}{P_a} \right)^3 \right] A_{cr}$$

ACI

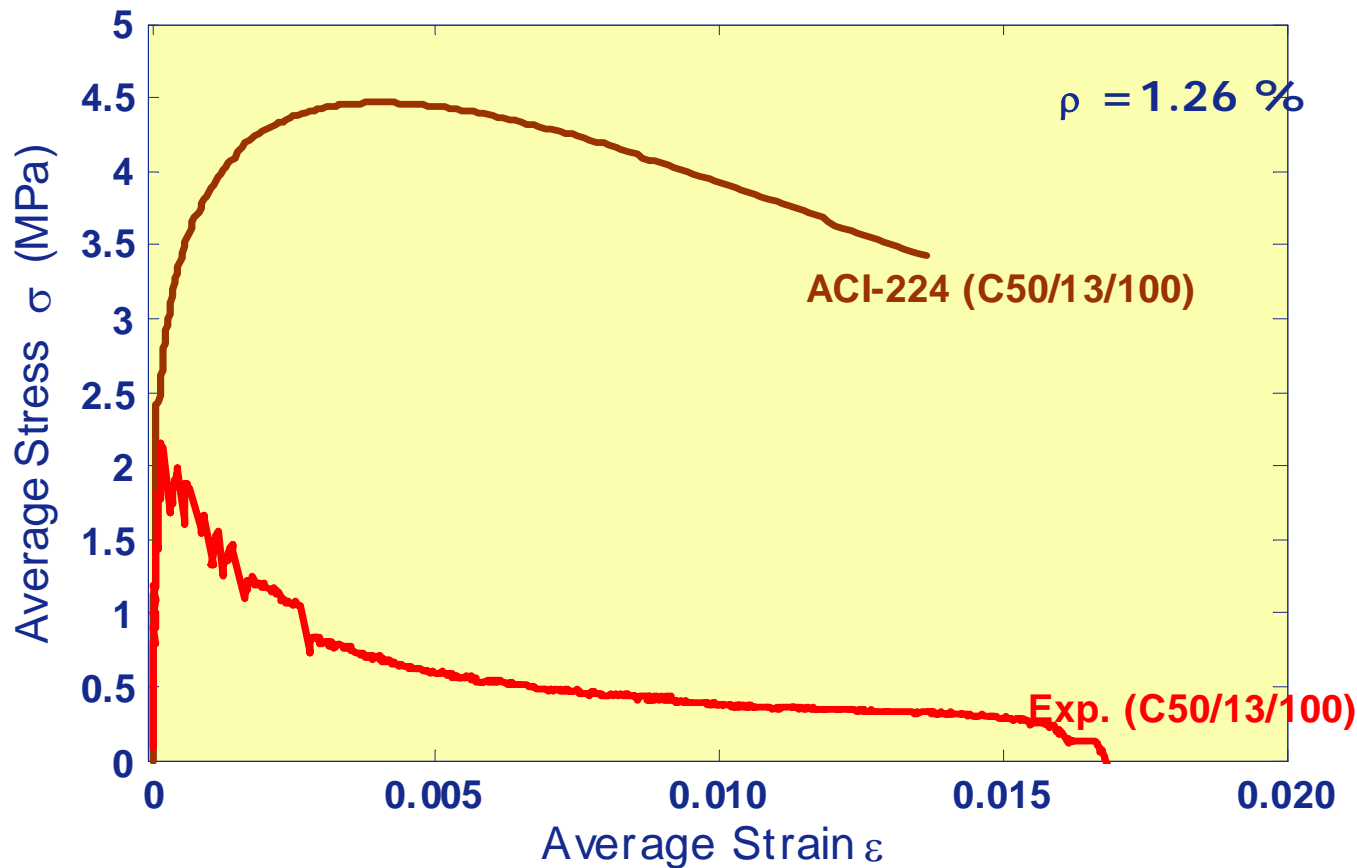
Modified to account for weak FRP bond 
$$A_e = \left[ \frac{P_{cr}}{P_a} \right]^3 \beta_d A_g + \left[ 1 - \left( \frac{P_{cr}}{P_a} \right)^3 \right] A_{cr}$$

## Composite strain for given bar strain

$$\epsilon_m = \epsilon_s \left[ 1 - K \left( \frac{f_{scr}}{f_f} \right)^2 \right]$$

CEB

$$f_{scr} = \frac{P_{cr}}{A_f} = f'_t \left( \frac{1}{\rho} - 1 + n_f \right)$$



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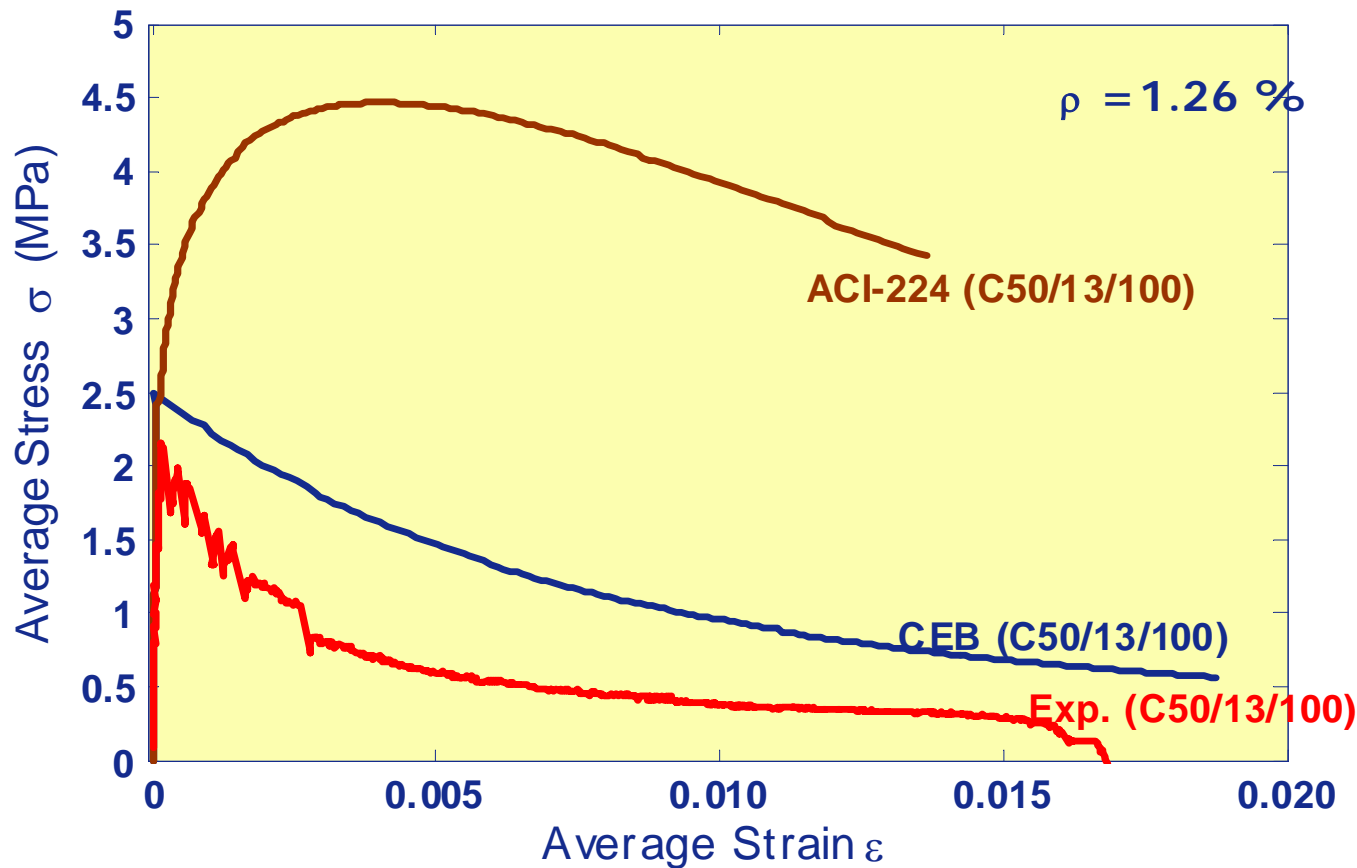
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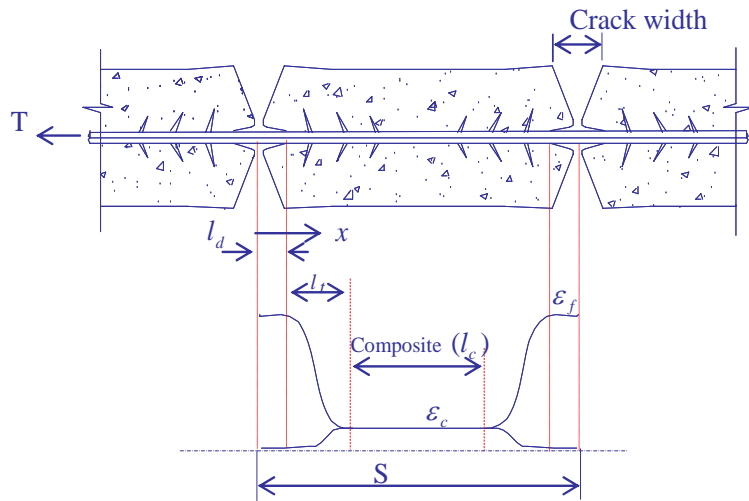
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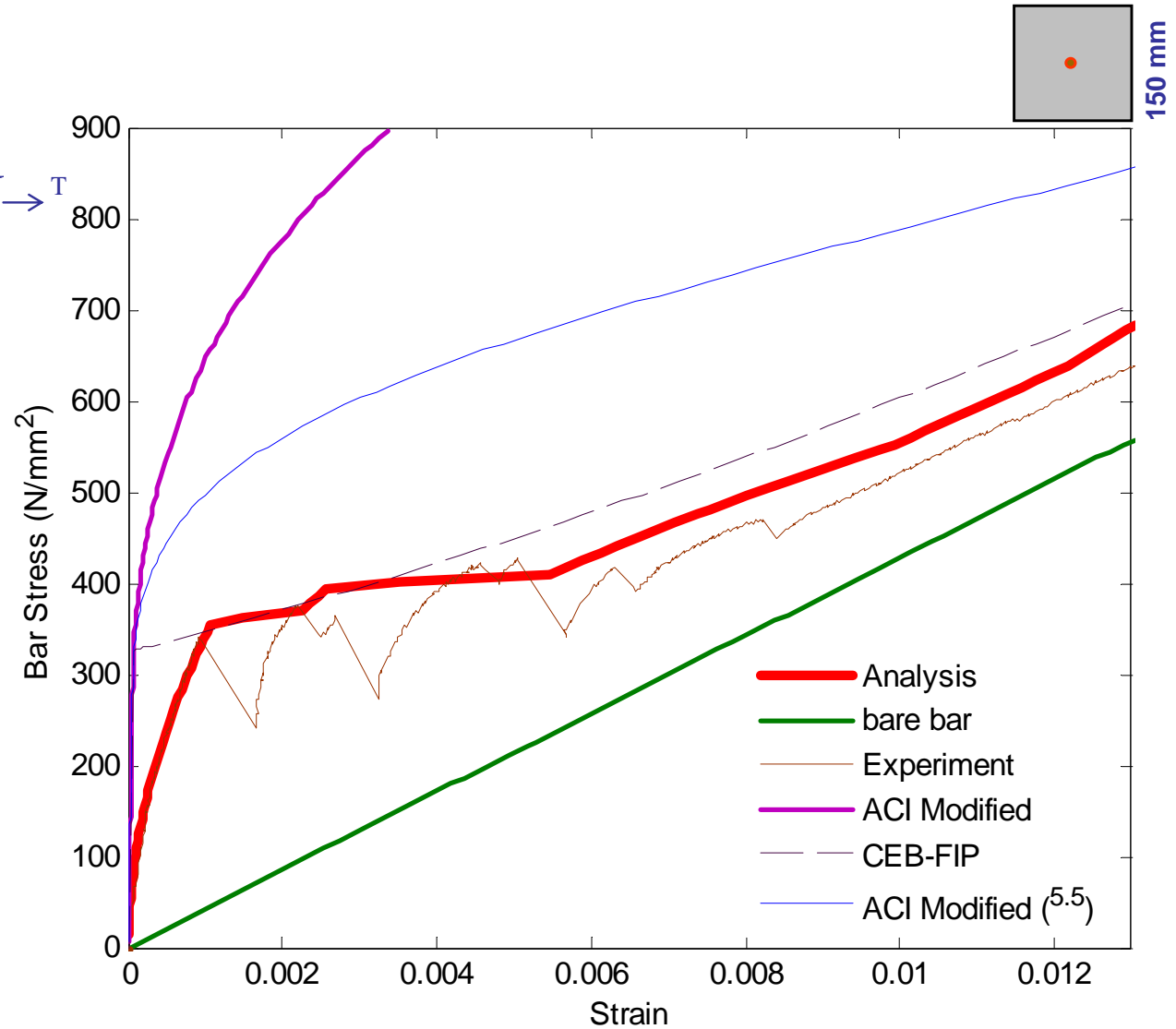
$$f_{scr} = \frac{P_{cr}}{A_f} = f'_t \left( \frac{1}{\rho} - 1 + n_f \right)$$



$$\epsilon_s = \left( \frac{p}{A_s E_s} \right)$$

$$\epsilon_s(x) = 0.5 \left( \cos \left( \frac{\pi x}{l_t} \right) + 1 \right) (\epsilon_s - \epsilon_c) + \epsilon_c$$

$$\epsilon_s(x) = \epsilon_c(x) = \left( \frac{p}{A_s E_s + A_c E_c} \right)$$



Experimental result of 13mm bar in 150 square section compared with various models

- Existing tension stiffening models unconservative for GFRP-RC
- Deflections of GFRP-RC underestimated
- Three distinctive stages of tension stiffening behaviour corresponding to different stages of cracking identified.
- Substantial loss of composite action and early stages of bond deterioration in GFRP-RC
- Reinforcement ratio and concrete strength have direct influence on tension stiffening behaviour of GFRP-RC but not bar diameter
- Strain distribution functions used for modelling tension stiffening behaviour of GFRP-RC.