Flexural and Punching Shear Design of FRP RC Flat Slabs

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Outline

- Context of Work
- Experimental Program
- Finite Element Verification
- Parametric Study
- Flexural Design Guidelines
- Punching Shear Design Guidelines
- Conclusions
Context Of Work

• Eurocrete Project
  • **Aim:** Durable FRP Reinforcement
  • **Partners:** 9 Companies + University of Sheffield
  • **Funds:** 5.6 million ECU

• *fib* TG 9.3 + ConFibreCrete Research Network
  • **Aim:** Design Guidelines
  • **Members:** Over 40 International Experts
  • **TMR:** 11 Institutions from 9 EC Countries
  • **Funds:** 1.3 million Euro
Work Done @ Sheffield (on FRP Rebar)

- RC Beams: Flexure, Shear, Bond & Ductility
- RC Slabs: 2-D Flexure, Punching Shear, Bond
- Sub-elements: Tensile strength, Bond
- Development of Design Guides and Philosophy
- Various Case Studies on Precast Concrete Elements
- Design of New Structural Elements / Applications
Work on FRP RC Slabs

EXPERIMENTAL PROGRAM

Series (1)
Flat slab testing

Phase (1)
S = 200 mm
No shear
SG1
SC1
Shear
SGS1
SCS1

Phase (2)
S = 100 mm
No shear
SG2 & 3
SC2
Shear
SGS2

Series (2)
Ground anchor slab testing

Phase (1)
Square slabs
Steel

Phase (2)
Circular slabs
GFRP
Hoop
Spiral
TGS-S1 to 3
TGS-G1 to 3
TGS-A to D
TAS-1 to 6

ANALYTICAL PROGRAM
(ABAQUS)

Comparisons with Test Results
Parametric Study
Experimental Program
Slab Details

First Series

Without Shear Reinforcement

With Shear Reinforcement

Second Series

Without Shear Reinforcement

With Shear Reinforcement

Typical CFRP ‘Shearband’
FEA Sensitivity Study

- Element Type
- Mesh Sensitivity
- Integration Points
- Support Conditions
- Reinforcement Model
- Concrete Model
  - Uniaxial Compressive Stress-Strain Curve
  - Initial Compressive Young’s Modulus
  - Ratio of Uniaxial Comp. to Tensile Strength
  - Tension Stiffening
  - Shear Retention

Mesh (C): 20 × 20 elem.
FEA Verification
Comparisons with Test Results

Slab SG2

Load (kN) vs. Displacement D850 (mm)

- Blue line: test
- Red line: analysis
Parametric Study
Reinforcement Ratio

Expected Failure Trajectory

- $f_{cu} = 60 \text{ MPa}$
- $L/t = 23$

Load (kN) vs. Displacement $\Delta 50$ (mm)

- Glass
- Carbon

Reinforcement Ratios:
- $\rho = 3.6\%$
- $\rho = 3\%$
- $\rho = 1.8\%$
- $\rho = 0.9\%$
- $\rho = 0.45\%$
- $\rho = 0.225\%$
Parametric Study
Concrete Grade

Load (kN) vs. Displacement \( \delta \) (mm)

- \( \rho = 0.9\% \)
- \( L/t = 23 \)

Expected Failure Trajectory for Carbon

- \( f_{cu} = 60 \)
- \( f_{cu} = 50 \)
- \( f_{cu} = 35 \)
- \( f_{cu} = 25 \)

Expected Failure Trajectory for Glass

- \( f_{cu} = 60 \)
- \( f_{cu} = 50 \)
- \( f_{cu} = 35 \)
- \( f_{cu} = 25 \)
Parametric Study
Span to Depth Ratio

Load (kN)

Displacement D50 (mm)

\[ \rho = 0.9 \% \]
\[ f_{cu} = 60 \text{ MPa} \]

Expected Failure Trajectory for Carbon

Expected Failure Trajectory for Glass

\( L/t = 15 \)
\( L/t = 20 \)
\( L/t = 23 \)
\( L/t = 30 \)
\( L/t = 35 \)

Glass
Carbon
Flexural Guidelines

Design Equation

\[(L/t) = 24.2 \times (250/\Delta r)^{1.9} \times (E\rho/99000)^{0.1} \times (f_{cu}/40)^{-0.44}\]

- Parametric Study CFRP RC Slabs
- Ultimate Loads
- Service Loads = Ultimate Loads / 1.6

Service Loads + FEA Load-Deflection curves → Service Deflections \(\delta_{ser}\)

\[\delta_{ser} = L / \Delta r\]
Flexural Guidelines
Derivation of Design Equation

$y = 0.928 x^{0.0546}$

$R^2 = 0.9814$
Flexural Guidelines
Derivation of Design Equation

\[ y = 1.0406 x^{-0.2331} \]

\[ R^2 = 0.9167 \]
Flexural Guidelines

Derivation of Design Equation

\[
(\Delta r/250) = 0.918 (E_p/99000)^{0.0546} (f_{cu}/40)^{-0.2331} [(L/t)/23]^{-0.5547}
\]

\[
y = 0.9506 x^{-0.5547} \\
R^2 = 0.9914
\]
Flexural Guidelines

Derivation of Design Equation

\[
(L/t) = C \left( \frac{250}{\Delta r} \right)^{c_1} \left( \frac{E\rho}{99000} \right)^{c_2} \left( \frac{f_{cu}}{40} \right)^{c_3}
\]

\[
y = 0.9995 \times x
\]

\[
R^2 = 0.9585
\]

\[
(L/t) = 24.2 \left( \frac{250}{\Delta r} \right)^{1.9} \left( \frac{E\rho}{99000} \right)^{0.1} \left( \frac{f_{cu}}{40} \right)^{-0.44}
\]
Punching Shear Guidelines
Failure Patterns (2nd Series)

Section through Slab SG3 at Failure

Section through Slab SC2 at Failure

Section through Slab SGS2 at Failure
Strain Approach

\[ v_c = (100 \frac{A_e}{b_v d})^{1/3} (400 / d)^{1/4} (0.27) (f_{cu})^{1/3} , \]

\[ A_e = A_{FRP} \left( \frac{E_{FRP}}{E_{steel}} \right) \]

Modified Approach

\[ v_c = (100 \frac{A_e}{b_v d})^{1/3} (400 / d)^{1/4} (0.27) (f_{cu})^{1/3} , \]

\[ A_e = A_{FRP} \left( \frac{E_{FRP}}{E_{steel}} \right) (\phi) \]

\[ \varepsilon = \varepsilon_{FRP} / \varepsilon_{yield\ steel} \]

when \( \varepsilon_{FRP} = 0.0045, \phi = 1.8 \)
**Punching Shear Guidelines**

Concrete Shear Resistance

Normalised Exp. & Pred. Capacities (BS 8110)

**Strain Approach**

\[ v_c = \left(100 \frac{Ae}{ bv \; d}\right)^{1/3} \left(\frac{400}{d}\right)^{1/4} \left(0.27\right) \left(\frac{fcu}{25}\right)^{0.33}, \text{ where } Ae = Ar \left(\frac{Er}{200}\right) \]

**Modified Approach**

\[ v_c = \left(100 \frac{Ae}{ bv \; d}\right)^{1/3} \left(\frac{400}{d}\right)^{1/4} \left(0.27\right) \left(\frac{fcu}{25}\right)^{0.33}, \text{ where } Ae = Ar \left(1.8Er/200\right) \]
Punching Shear Guidelines
Concrete Shear Resistance
Verification of the Modified Approach (BS 8110)

MODIFIED APPROACH (BS 8110)

STRAIN APPROACH (BS 8110)
CONCLUSIONS

• Flexural Design Guidelines
  • FEA can predict flexural deformations
    (though convergence problems encountered)
  • Main parameters influencing deflections \( (L/t, E, \rho, f_{cu}) \)
  • General Design Equation proposed \[ L/t = f(\Delta r, E, \rho, f_{cu}) \]

• Punching Shear Design Guidelines
  • Strain App. is conservative
  • Modified App. offers good predictions \( (\phi = 1.8) \)