Tension Stiffening in GFRP RC Beams

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Tension Stiffening in GFRP RC Beams

OUTLINE

- Background
- Methodology
- Experimental Programme
- Experimental Results
- Conclusions
Serviceability-controlled design

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\[ I_e = I_{cr} + \left( \beta_d I_g - I_{cr} \right) \left[ \frac{M_{cr}}{M_a} \right]^3 \leq I_g \]

\[ \beta_d = \alpha_b \left[ \frac{E_f}{E_s} + 1 \right] \]

\[ \alpha_b = \text{bond coefficient} = 0.5 \]

ISTructE (1999)
- Steel RC equations applicable
BACKGROUND

Key Variables
- Reinforcement Ratio
- Modulus of Elasticity
- Bond Characteristics

Other Variables
- Concrete cover
- Concrete Strength
- Rebar diameter
Tension Stiffening in GFRP RC Beams

**BACKGROUND**

Load, $N$

1 = uncracked stage
2 = crack formation stage
3 = stabilized cracking stage
4 = yield stage
5 = bare steel rebar

$\frac{f_{ctm} \alpha_c}{E_s}$

$\beta \frac{f_{ctm}}{E_s \rho}$

$\frac{1.3N_{cr}}{N_{cr}}$

$f_{ctm}$ = tensile strength of concrete
$E_s$ = steel modulus
$E_c$ = concrete modulus
$\alpha_e$ = modular ratio = $E_s / E_c$

Load-strain relationship of a steel RC tie *(fib 1999)*
Load-strain relationship of a steel RC tie (*fib 1999*)

- **Load, N**: 1 = uncracked stage, 2 = crack formation stage, 3 = stabilized cracking stage, 4 = yield stage, 5 = bare steel rebar
- **Ncr** = tensile strength of concrete
- **Es** = steel modulus
- **Ec** = concrete modulus
- **αe** = modular ratio = $E_s / E_c$

**Equations**:

- $f_{ctm} = \frac{f_{ctm}(0.6+\alpha_e\rho)}{E_s\rho}$
- $f_{ctm} = \frac{\beta f_{ctm}}{E_s\rho}$
- $\rho = \frac{A_s}{A_c}$
- Average strain, $\frac{\Delta L}{L}$

**Diagram**:
- Load vs. strain graph
- Stages: uncracked, crack formation, stabilized cracking, yield, bare steel rebar
Tension Stiffening in GFRP RC Beams

METHODOLOGY

- **Experimental:**
  - Structural tests: (Beams and Slabs, CFRP and GFRP)
  - Material tests: (Concrete and rebars)

- **Analytical:**
  - Sectional analysis
  - Finite element analysis
EXPERIMENTAL PROGRAMME (GFRP RC Beams)
## EXPERIMENTAL PROGRAMME (GFRP RC Beams)

<table>
<thead>
<tr>
<th>Rebar type</th>
<th>Series designation</th>
<th>Beam designation</th>
<th>Rebar details</th>
<th>Reinforcement ratio</th>
<th>Relation to control steel beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>BG1</td>
<td>BG1a, BG1b</td>
<td>2∅9.53</td>
<td>0.00432</td>
<td>Equal flexural capacity</td>
</tr>
<tr>
<td></td>
<td>BG2</td>
<td>BG2a, BG2b</td>
<td>2∅12.7</td>
<td>0.00772</td>
<td>Equal area of rebars</td>
</tr>
<tr>
<td></td>
<td>BG3</td>
<td>BG3a, BG3b</td>
<td>4∅19.05</td>
<td>0.0391</td>
<td>Equal stiffness of rebars</td>
</tr>
<tr>
<td>Steel</td>
<td>BS</td>
<td>BSa, BSb</td>
<td>2∅12</td>
<td>0.00688</td>
<td>-</td>
</tr>
</tbody>
</table>
**EXPERIMENTAL PROGRAMME (GFRP RC Beams)**

<table>
<thead>
<tr>
<th>Rebar type</th>
<th>Nominal diameter, (mm)</th>
<th>Modulus of elasticity, (MPa)</th>
<th>Tensile strength, (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>9.53</td>
<td>42500</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td>12.70</td>
<td>41500</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>19.05</td>
<td>42000</td>
<td>650</td>
</tr>
</tbody>
</table>

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July 3-7, 2006 – Alexandroupolis, Greece

Tension Stiffening in GFRP RC Beams
EXPERIMENTAL PROGRAMME (GFRP RC Beams)

- Strain Gauge on Rebar
- Strain Gauge on concrete

1/3 shear span

Detail of 10 strain gauges, No. (5-14), at midspan zone.

Midspan forced crack

Anticipated natural crack

Anticipated natural crack

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Tension Stiffening in GFRP RC Beams
EXPERIMENTAL PROGRAMME (GFRP RC Beams)
EXPERIMENTAL PROGRAMME (GFRP RC Beams)
EXPERIMENTAL RESULTS

Beam BG2a

- Strain Gauge 10
- Strain Gauge 9
- Strain Gauge 8
- Strain Gauge 7
- Strain Gauge 6
- Strain Gauge 5
- Cracked-section analysis

Crack Locations
- 80.0 kN
- 60.0 kN
- 40.0 kN
- 20.3 kN (just after adjacent crack)
- 14.0 kN (just after first crack)
- 13.3 kN (before cracking)

Load, kN

Concrete Strain, microstrain

Curvature, m

Curvature derived from experimental strains

Cracked-section analysis

Tension Stiffening in GFRP RC Beams

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The University Of Sheffield.
EXPERIMENTAL RESULTS

Beam BG2a

- LVDT L6
- Cracked-section analysis
- Flexural deflections derived from experimental curvatures

Load, kN

Midspan Deflection, mm

Stress at crack, in MPa

average strain between cracks

strain at crack

Distance From Support, mm

Average Bond Stress, MPA

Average Bond, MPa

Stabilized cracking phase

two cracks adjacent to crack inducer

1st crack at crack inducer

Average strain, microstrain
EXPERIMENTAL RESULTS

Load = 20.8 kN
Average crack width = 0.13 mm
Standard Deviation = 0.06

Load = 31.8 kN
Average crack width = 0.19 mm
Standard Deviation = 0.06

Load = 47.2 kN
Average crack width = 0.27 mm
Standard Deviation = 0.1

Load = 74.7 kN
Average crack width = 0.4 mm
Standard Deviation = 0.1

Note: Crack widths in flexure zone, at the bottom concrete fibre.
EXPERIMENTAL RESULTS

Beams BG, BS

Load [kN] vs. Midspan Deflection [mm] graph showing load-deflection curves for various beams BG and BS with different reinforcement ratios.

- $\rho_f = 0.039$
- $\rho_s = 0.0068$
- $\rho_f = 0.0077$
- $\rho_f = 0.0043$

Key:
- BG1a
- BG1b
- BG2a
- BG2b
- BG3a
- BG3b
- BSa
- BSb
TENSION STIFFENING ANALYSIS

Beam Series BG

Stress at crack, Mpa

Average strain, microstrain

BG1b
BG2a
BG3b
naked 9.53 mm rebar
naked 12.7 mm rebar
naked 19 mm rebar
## TENSION STIFFENING ANALYSIS

### Tension Stiffening in the Stabilised Cracking Phase

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>Naked rebar modulus, $E_f$, “MPa”</th>
<th>Effective Modulus, $E_{eff}$, “MPa”</th>
<th>$E_{eff}/E_f$</th>
<th>Effective Modulus, $E_{eff}$, “MPa”</th>
<th>$E_{eff}/E_f$</th>
<th>Average $E_{eff}/E_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BG3b</td>
<td>41970</td>
<td>47715</td>
<td>1.14</td>
<td>44548</td>
<td>1.06</td>
<td>1.1</td>
</tr>
<tr>
<td>BG2a</td>
<td>41600</td>
<td>49889</td>
<td>1.20</td>
<td>44636</td>
<td>1.07</td>
<td>1.14</td>
</tr>
<tr>
<td>BG1b</td>
<td>42750</td>
<td>492280</td>
<td>1.15</td>
<td>45370</td>
<td>1.06</td>
<td>1.11</td>
</tr>
</tbody>
</table>

For steel RC, $\rho$=1%

|                          | $E_{eff}/E_f$ | 1.6 | 1.39 |

For steel RC, $\rho$=10%

|                          | $E_{eff}/E_f$ | 1.31 | 1.03 |
**TENSION STIFFENING ANALYSIS**

\[ f_{ctm} = \text{concrete tensile strength} \]
\[ E_f = \text{FRP rebar elastic modulus} \]
\[ E_c = \text{concrete elastic modulus} \]
\[ \alpha_e = \text{modular ratio} = \frac{E_f}{E_c} \]
\[ N_{cr} = \text{cracking load} \]
\[ \rho = \frac{A_f}{A_c} \]

Load, \(N\)

1 = uncracked stage
2 = crack formation stage
3 = stabilized cracking stage
4 = naked rebar

\[ f_{ctm}\alpha_e = \frac{E_f}{N} \]

\[ 1.3N_{cr} \]

\[ N_{cr} \]

\[ E_{eff} = \text{effective rebar modulus} \]

\[ 1.2E_f \text{ at point A} \]

\[ 1.05E_f \text{ at point B} \]

Proposed load-strain relationship of a GFRP RC tension tie
TENSION STIFFENING ANALYSIS

Tests of GFRP RC tension ties by Sooriyaararachchi
TENSION STIFFENING ANALYSIS

Midspan Deflection of Beam BG3

Load, kN

Experimental / LVDT L6
Proposed relationship

Midspan Deflection, mm

Midspan Deflection of Beam BG3
Cracked-section analysis

FE Analysis

Cracked-section analysis

FE Analysis

Crack Locations

Crack Locations

Crack Locations

Crack Locations

Crack Locations

Crack Locations

Finite Element Analysis

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Finite Element Analysis
CONCLUSIONS

Behaviour of GFRP RC beams:

- Deflection of GFRP RC is mainly caused by flexural curvatures.
- Shear-induced deflections may not be negligible for low reinforcement ratios and deep-penetrating wide cracks.
- The response of the compressive concrete zone requires further consideration due to the increased localised effect of cracks.
- GFRP RC show good bond.
- CSA predicts the maximum rebar strain at a crack, underestimates the extreme-fibre concrete strain, and when shear induced deformations are sizeable may not provide an upper-bound deflection.
CONCLUSIONS

Tension Stiffening of GFRP RC beams:

- Contrary to steel RC, the amount of GFRP reinforcement may have negligible effect on tension stiffening in the stabilised cracking phase \((\rho > 0.4\%, \text{ or } \rho_{\text{eff}} > 1.25\%)\).  

- An average \(E_{\text{eff}} (1.10 \ E)\) offers a reasonable approximation for the whole stabilised cracking phase.

- Tension stiffening in GFRP is lower than steel RC, which is due to the high levels of rebar strain involved.

- An initial tension stiffening relationship is proposed but needs to be expressed more fundamentally.
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